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On acceleration of Evolutionary Algorithms solution process applied to large, non-liner, constraint optimization problems

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CONTENTS

1. Introduction

Research motivation

Solution of large, non-linear, constraint optimization problems,

especially engineering ones, e.g.:

1. Residual stress analysis in railroad rails and vehicle wheels.
2. Physically based approximation of experimental data.

Research objective

Significant acceleration of optimization process based on:

1. A choice of the best combination of evolutionary operators (including various benchmark tests and various evaluation methods).
2. Use of several simple acceleration techniques proposed here;
   some of them are addressed to specific types of optimization problems,
   where an unknown function is searched

2. Benchmark problems

Benchmark tests selection criteria:

- number of decision variables
- dimension of physical solution space
- convex/non-convex fitness functions and/or feasible region
- number of local and global extreme points
- smoothness of the fitness function
- ratio of the number of equality and inequality constraints to the number of decision variables
- size of feasible region

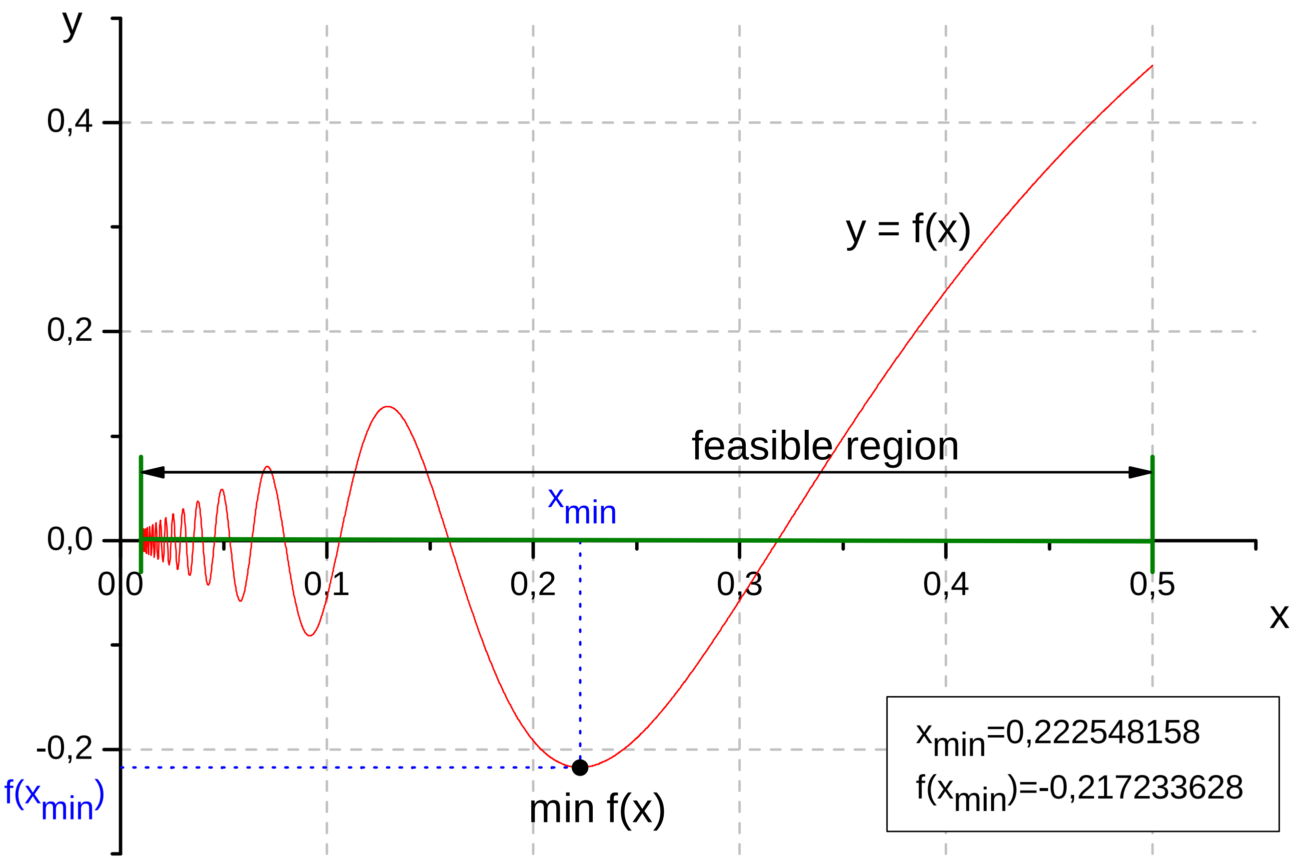
Benchmark test (i)

Find

$$\min_x x\sin\left(\frac{1}{x}\right)$$

where

$$x \in [0{,}01; 0{,}5]$$



Benchmark test (ii)

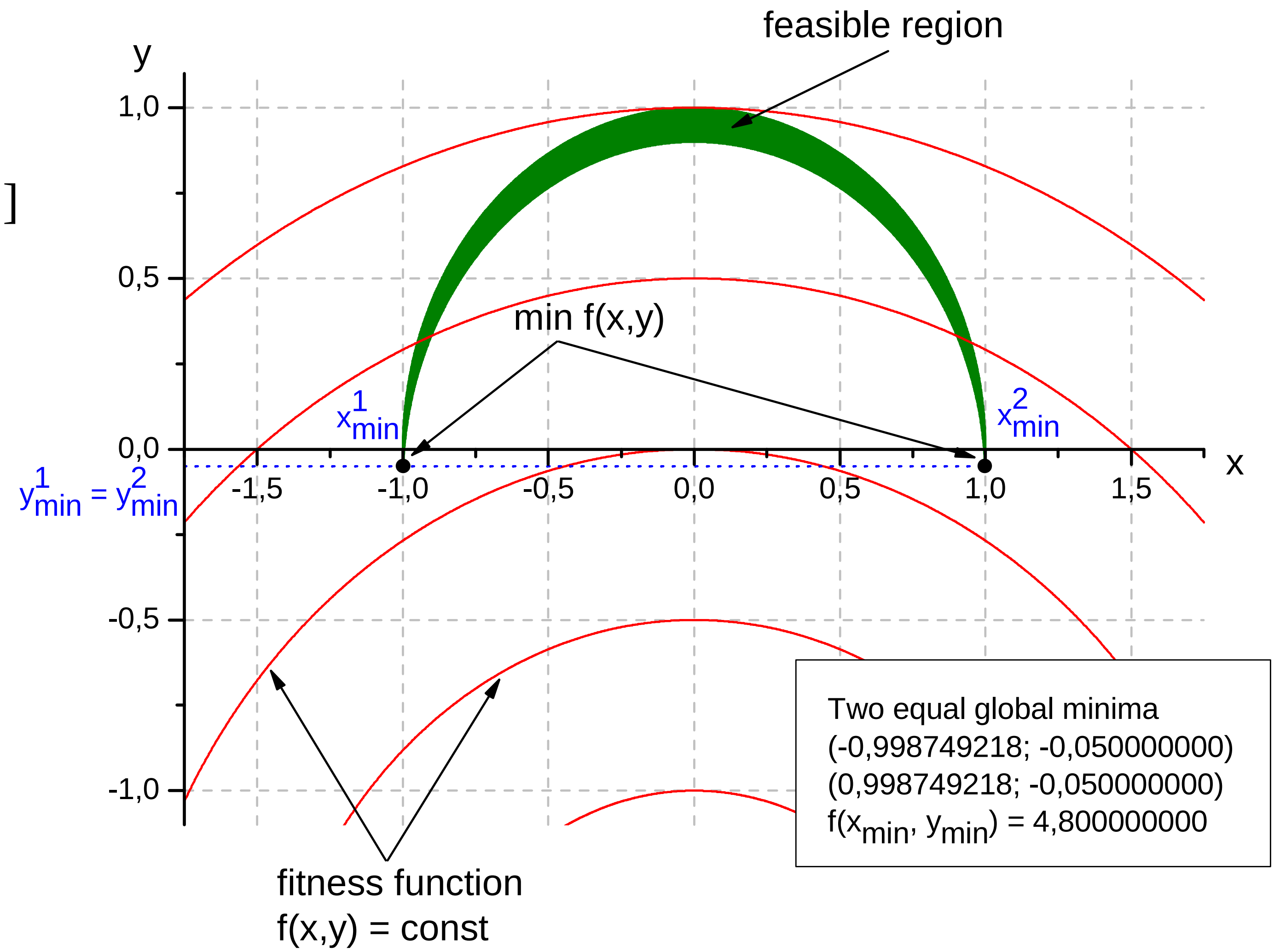
Find

$$\min_{x,y}[x^2 + (y+2)^2]$$

at constraints

$$x^2 + y^2 \le 1$$

$$x^2 + (y+0{,}1)^2 \ge 1$$



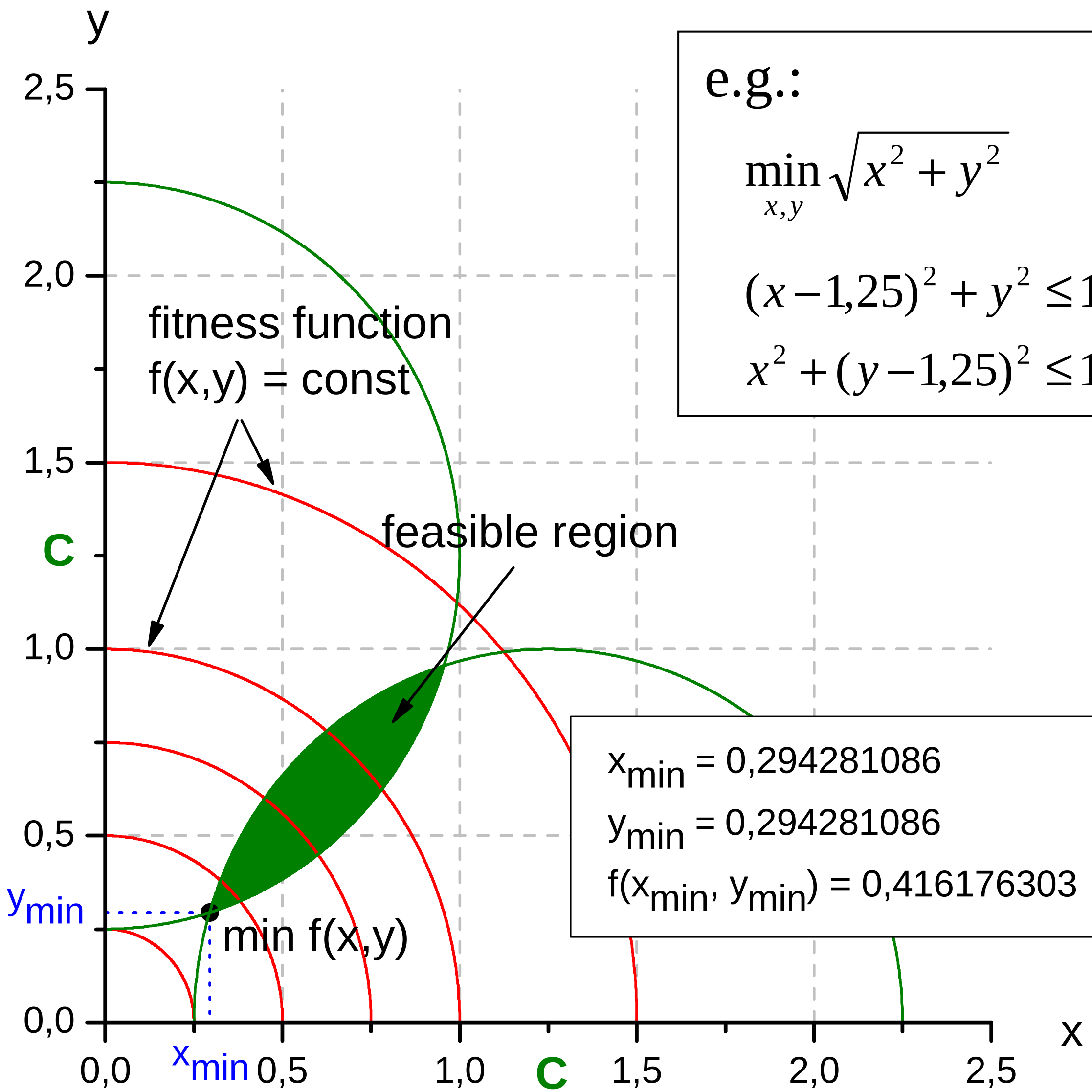
Benchmark test (iii)

Find

$$\min_{x_1, x_2, \ldots, x_N} \sqrt{\sum_{i=1}^{N} x_i^2}$$

at constraints

$$(x_1 - C)^2 + x_2^2 + \ldots + x_N^2 \le 1$$
$$x_1^2 + (x_2 - C)^2 + \ldots + x_N^2 \le 1$$
$$\vdots$$
$$x_1^2 + x_2^2 + \ldots + (x_N - C)^2 \le 1$$



e.g.:

$$\min_{x,y} \sqrt{x^2 + y^2}$$
$$(x - 1{,}25)^2 + y^2 \le 1$$
$$x^2 + (y - 1{,}25)^2 \le 1$$

x_{min} = 0,294281086
y_{min} = 0,294281086
$f(x_{min}, y_{min})$ = 0,416176303

Benchmark test (iv)

Find

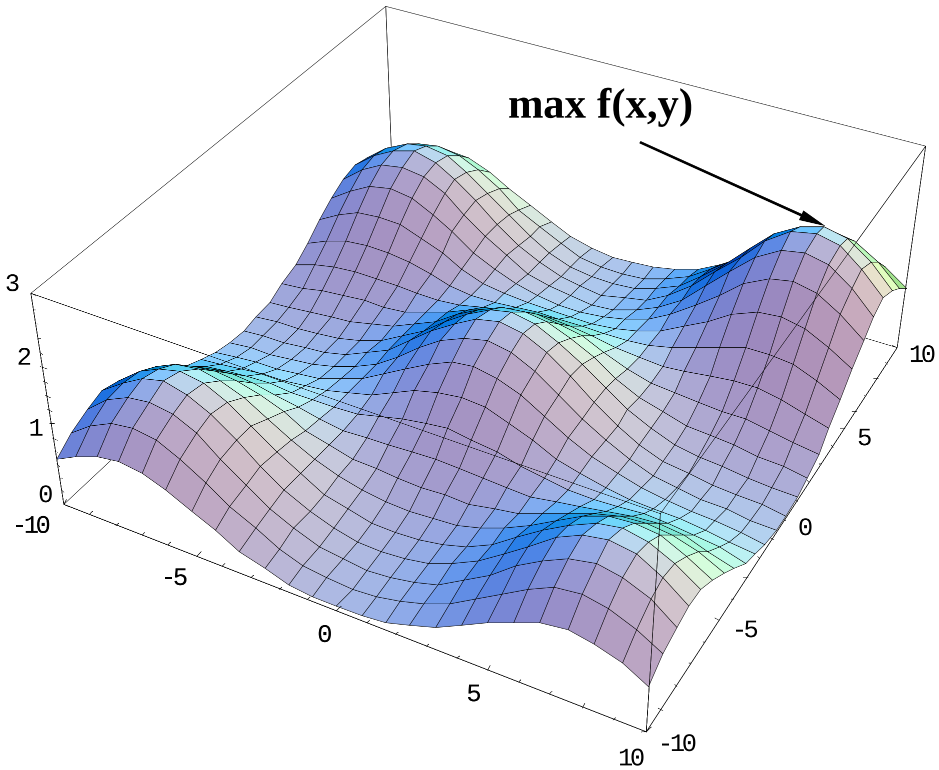
$$\max_{x,y} \sum_{i=1}^{n} f_i(x,y)$$

where

$$f_i(x,y) = \alpha_i \exp\left[-\left(\frac{x-\tilde{x}_i}{\beta_i}\right)^2 - \left(\frac{y-\tilde{y}_i}{\gamma_i}\right)^2\right]$$

$$(x,y) \in [x_1;x_2] \times [y_1;y_2]$$

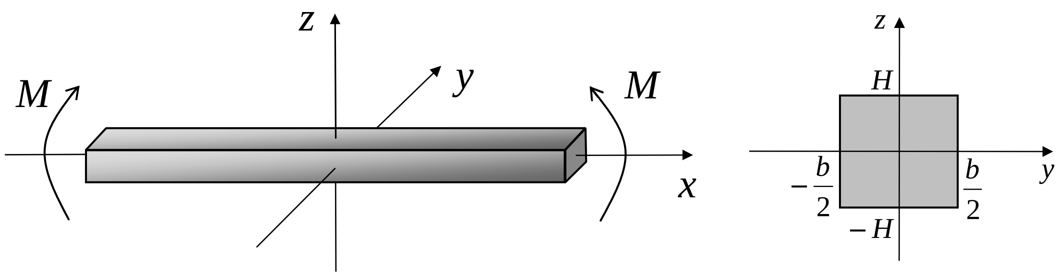
e.g.:



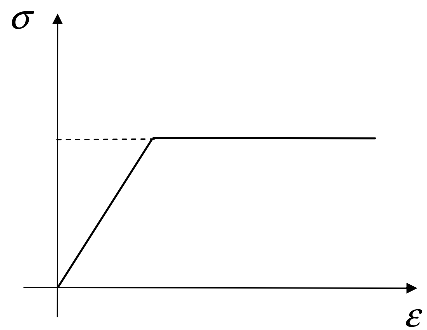
Benchmark test (v) – engineering problem:

residual stress analysis in a bar subject to cyclic bending

Pure cyclic bending:



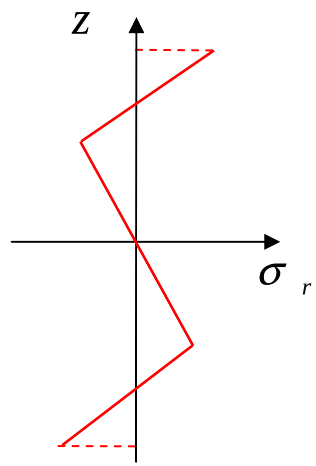
Elastic - perfectly plastic material:



Main features of the task:

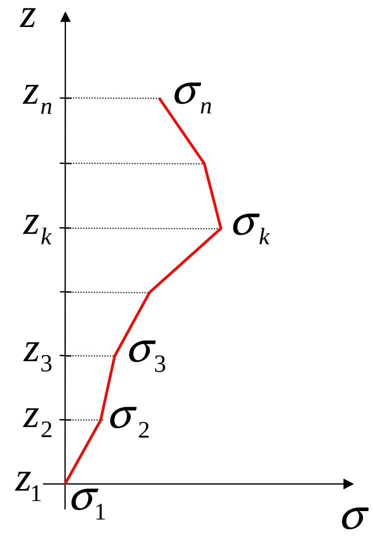
- Formulated as constraint optimization problem,
- May be formulated either as 1D or as 2D problem,
- Number of decision variables may be chosen,
- Exact solution is known.

Exact solution:



Final discrete 1D formulation of the problem:

Find stresses $\sigma_1, \sigma_2, \ldots, \sigma_n$ satisfying:



$$\min_{\sigma_1,\ldots,\sigma_{n-1}} \left(\sum_{k=1}^{n-1} \sigma_k^2 + \frac{1}{2}\sigma_n^2 \right)$$

minimum of total complementary energy

$$\sigma_n = -\frac{2}{z_n} \sum_{k=1}^{n-1} \sigma_k z_k$$

global equilibrium equation

$$-1 \le \frac{\sigma_k}{\sigma_Y} - \frac{3}{2}\left[1 - \frac{1}{3}\left(\frac{\bar{Z}}{H}\right)^2\right]\frac{k-1}{n-1} \le 1$$

condition for total stresses (plastic limit)

$$k = 1, \ldots, n$$

In calculation: $\sigma_Y = 1 \qquad \dfrac{\bar{Z}}{H} = \dfrac{1}{2}$

3. On results evaluation

Criteria:

- the error after *n* generations
- convergence rate
- effectivity factor (percentage of successful results)

Classification type (”1” is the best):

- „natural”	1,2,3,…	C1
- „Olympic” (Fibonacci series)	1,2,3,5,8,13,…	C2
- weighted multi-criteria		C3

4. Choice of the best combination of evolutionary operators

Combination of operators	Convergence rate (1)	True error (2)	Successful tests [%] (3)	Classification of			Classification due to		
				(1)	(2)	(3)	C1	C2	C3
tournament, heuristic, uniform	-0,597837787	1,10E-06	99	11	14	2	5	5	5
tournament, heuristic, non-uniform	-0,598647339	1,09E-06	100	10	13	1	6	4	6
tournament, heuristic, boundary	-0,603888971	1,47E-06	98	9	15	3	7	7	7
tournament, arithmetic, uniform	-0,564716923	3,77E-07	49	15	8	15	16	16	14
tournament, arithmetic, non-uniform	-0,589256427	5,61E-07	53	12	10	14	13	13	13
tournament, arithmetic, boundary	-0,576161206	4,49E-07	48	13	9	16	15	14	16
ranking, heuristic, uniform	-0,728285685	7,80E-08	94	7	6	5	3	2	2
ranking, heuristic, non-uniform	-0,689088808	1E-06	91	8	12	6	2	3	3
ranking, heuristic, boundary	-0,793979245	9,47E-08	86	6	7	7	12	12	12
ranking, arithmetic, uniform	-0,870567421	6,19E-09	66	1	1	10	8	8	8
ranking, arithmetic, non-uniform	-0,845957396	1,28E-08	64	3	4	11	10	10	9
ranking, arithmetic, boundary	-0,825075728	7,99E-09	67	4	3	9	9	9	10
tournament, heuristic, uniform/non-un.	-0,561200132	3,01E-06	97	16	16	4	4	6	4
tournament, arithmetic, uniform/non-un.	-0,569744924	6,93E-07	55	14	11	13	14	15	15
ranking, heuristic, uniform/non-un.	**-0,863194509**	**1,81E-08**	**72**	**2**	**5**	**8**	**1**	**1**	**1**
ranking, arithmetic, uniform/non-uniform	-0,803757506	6,49E-09	61	5	2	12	11	11	11

Combination of operators	„Natural”	„Olympic”	Multi-criteria	Mean
tournament, heuristic, uniform / non-uniform	4	6	4	**4**
tournament, heuristic, uniform	5	5	5	**5**
tournament, heuristic, non-uniform	6	4	6	**6**
ranking, heuristic, non-uniform	2	3	3	**3**
ranking, heuristic, uniform	3	2	2	**2**
ranking, heuristic, uniform / non-uniform	1	1	1	**1**

5. Acceleration techniques proposed

- Mesh refinement

- Smoothing and balancing of raw EA solution

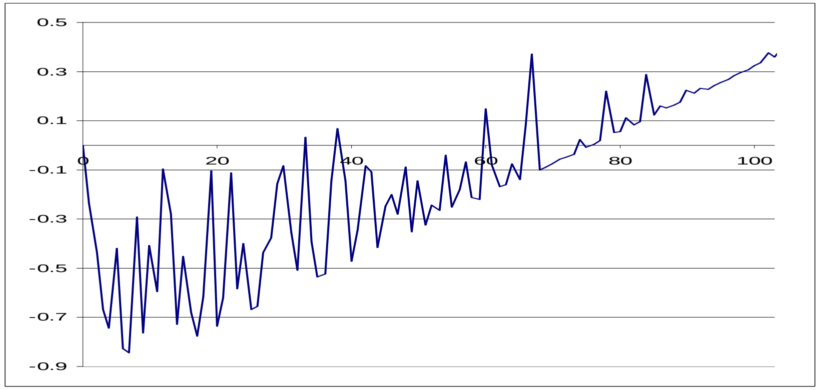
- A'posteriori error analysis, solution averaging, modification of evolutionary operators (concentration of calculations in zones of large errors)

- parallel and distributed calculations carried out on cluster

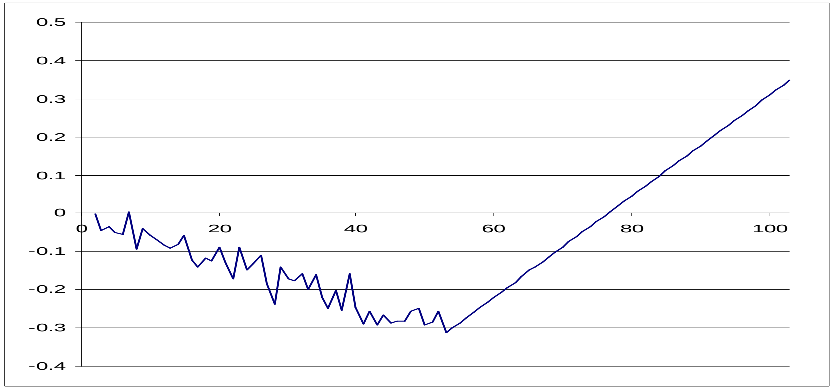
A choice of parameters and strategy of particular techniques.

Motivation example

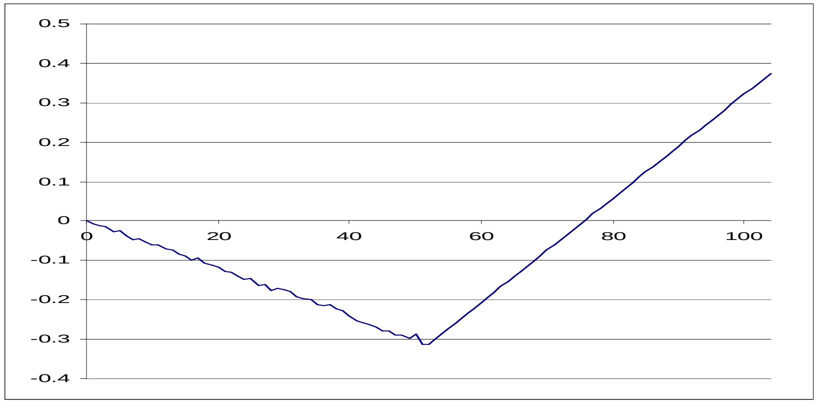
iterations number	fittness function
50	-24.86259



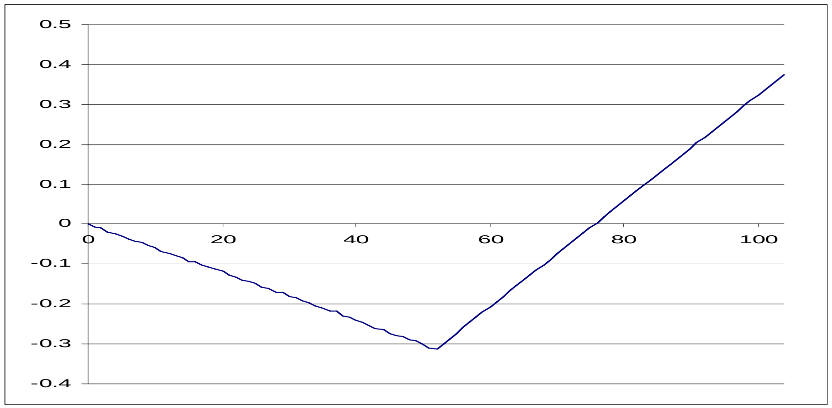
iterations number	fittness function
150	-7.10380



500	-7.00701



1000	-7.00514



Mesh refinement – an example

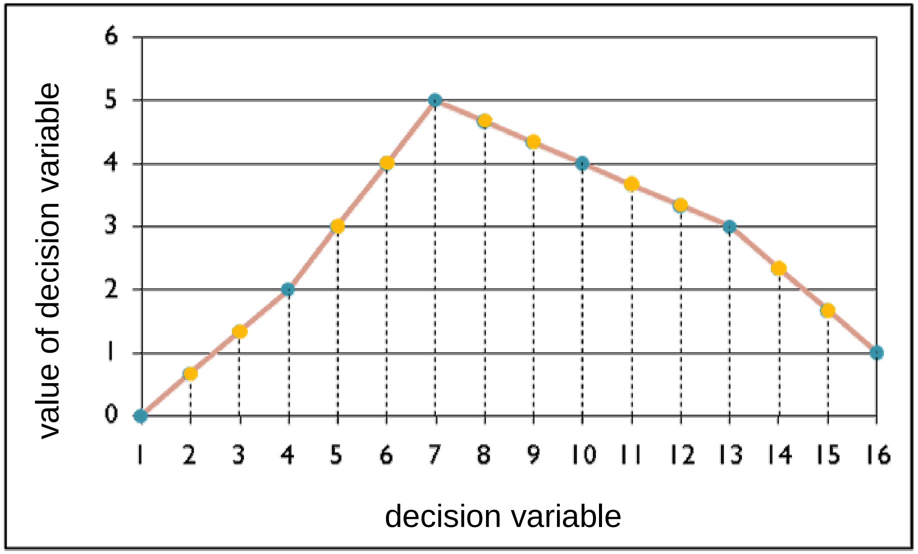
$$x_{n_1} = x_{s_1} + \Delta$$

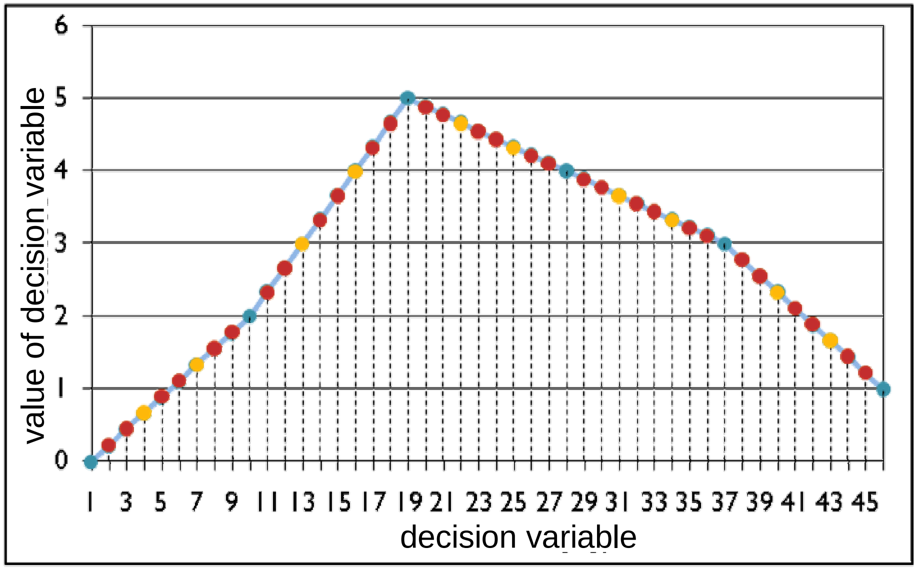
$$x_{n_2} = x_{s_1} + 2 \cdot \Delta = x_{s_2} - \Delta$$

$$\Delta = \frac{(x_{s_2} - x_{s_1})}{3}$$

$x_{n_1}\ x_{n_2}$ - values in the new nodes

$x_{s_1}\ x_{s_2}$ - values in the old nodes





$$n_n = n_s + (n_s - 1) \cdot 2$$

n_n - number of variables before refinement

n_s - number of variables after refinement

Smoothing by 1D MWLS approximation technique

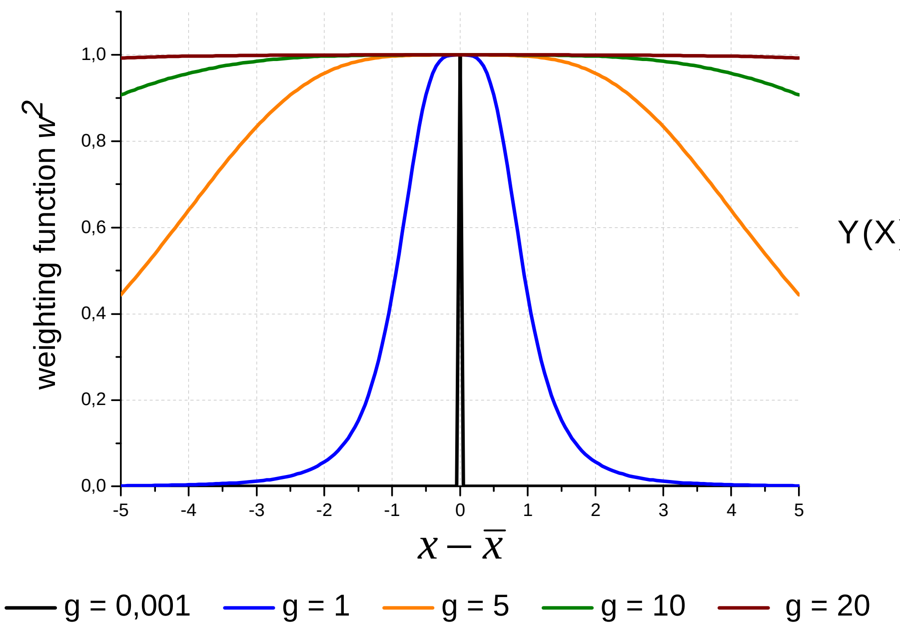
Weighted error functional: $B=\sum_{i=1}^{n}(u_i-\bar{u}_i)^2 w_i^2$, where $w_i^2=\left(h_i^2+\frac{g^4}{h_i^2+g^2}\right)^{-p-1}$ $h_i=\left|\bar{x}-x_i\right|$

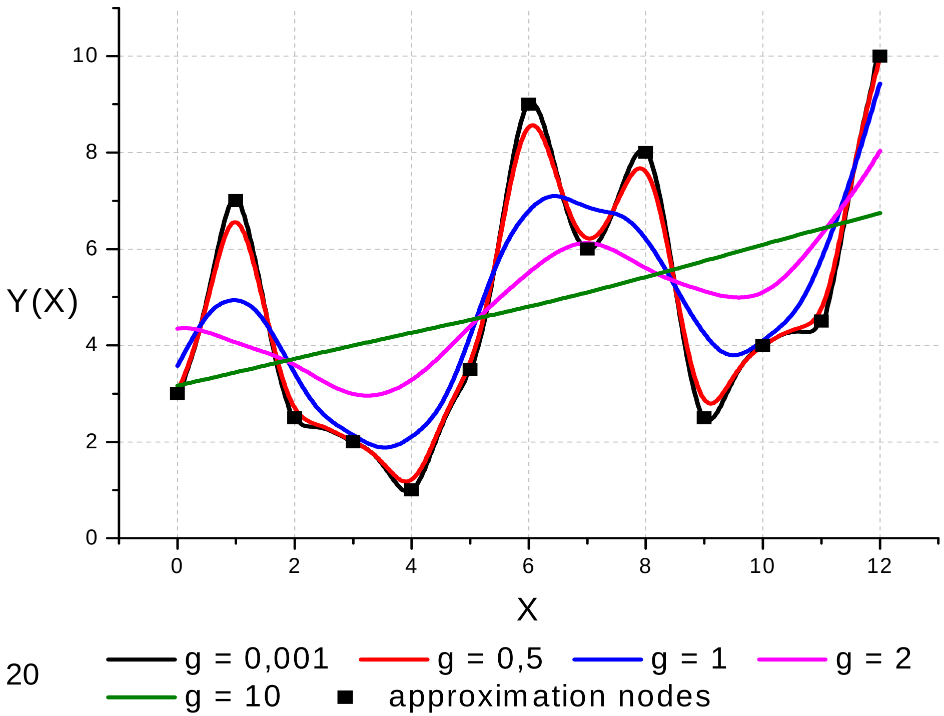
$$\bar{u}_i=\bar{u}(x_i)=\bar{u}+h_i\bar{u}'+\frac{1}{2}h_i^2\bar{u}''+\ldots+\frac{1}{p!}h_i^p\bar{u}^{(p)}+R \qquad \bar{u}=\bar{u}(\bar{x})$$

Minimization conditions: $\frac{\partial B}{\partial \bar{u}}=0,\quad \frac{\partial B}{\partial \bar{u}'}=0,\quad \frac{\partial B}{\partial \bar{u}''}=0,\quad \ldots,\quad \frac{\partial B}{\partial \bar{u}^{(p)}}=0$

$$\Rightarrow \boxed{\bar{u},\ \bar{u}',\ \bar{u}'',\ \ldots,\ \bar{u}^{(p)}}$$

Weighting function for various g values





Global equilibrium 2D balancing in the elastic-plastic beam subject to cyclic bending

Unbalanced resulting moments and axial force found upon raw solution σ_{raw}

$$M_Y(\sigma_{raw}) = \int x\sigma_{raw} d\Omega$$

$$M_X(\sigma_{raw}) = \int y\sigma_{raw} d\Omega$$

$$N(\sigma_{raw}) = \int \sigma_{raw} d\Omega$$

Balancing correction solution: $\Delta\sigma = ax + by + c$

Parameters *a, b, c* are found from the balance requirement:

$$\begin{cases} M_Y(\Delta\sigma) = M_Y(\sigma_{raw}) \\ M_X(\Delta\sigma) = M_X(\sigma_{raw}) \\ N(\Delta\sigma) = N(\sigma_{raw}) \end{cases} \Longrightarrow \quad a = \frac{M_Y(\sigma_{raw})}{I_Y} \quad b = \frac{M_X(\sigma_{raw})}{I_X} \quad c = \frac{N(\sigma_{raw})}{\Omega}$$

A'posteriori error estimation

Formulation:

Use simultaneously *m* independent populations
Find results

$$\left[z_1^1, z_2^1, z_3^1, \ldots, z_n^1\right]$$
$$\left[z_1^2, z_2^2, z_3^2, \ldots, z_n^2\right]$$
$$\left[z_1^3, z_2^3, z_3^3, \ldots, z_n^3\right]$$
$$\vdots$$
$$\left[z_1^m, z_2^m, z_3^m, \ldots, z_n^m\right]$$

where z_k^i - k-th decision variable from *i*-th solution

$i=1,2,\ldots,m$

$k=1,2,\ldots,n$

Calculate

(i) mean value

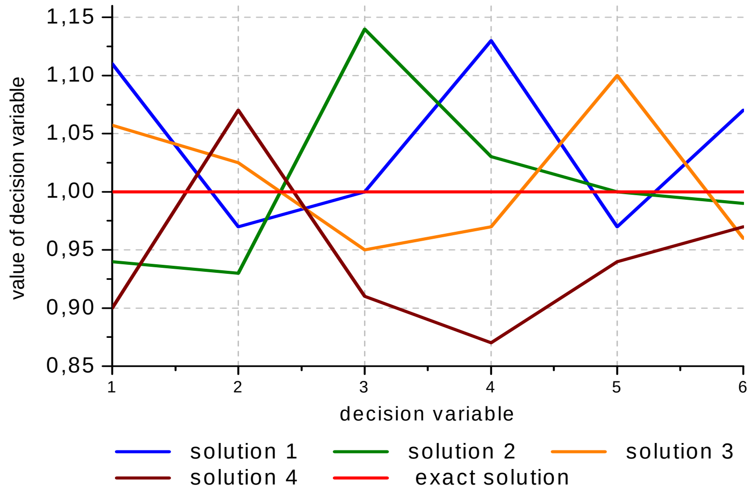
$$\bar{z}_k = \frac{1}{W}\sum_{i=1}^{m} \alpha_i z_k^i$$

where α_i - weighting factor, $W = \sum_{i=1}^{m} \alpha_i$

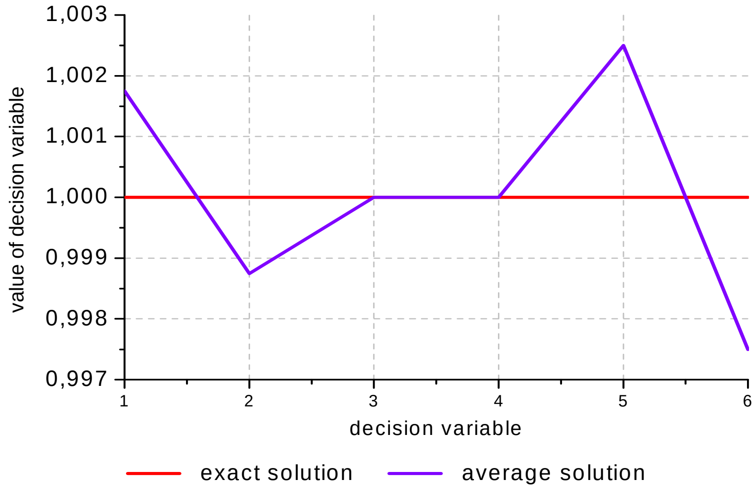
(ii) estimated error

$$E = [e_k^i] \qquad e_k^i = \left| z_k^i - \bar{z}_k \right|$$

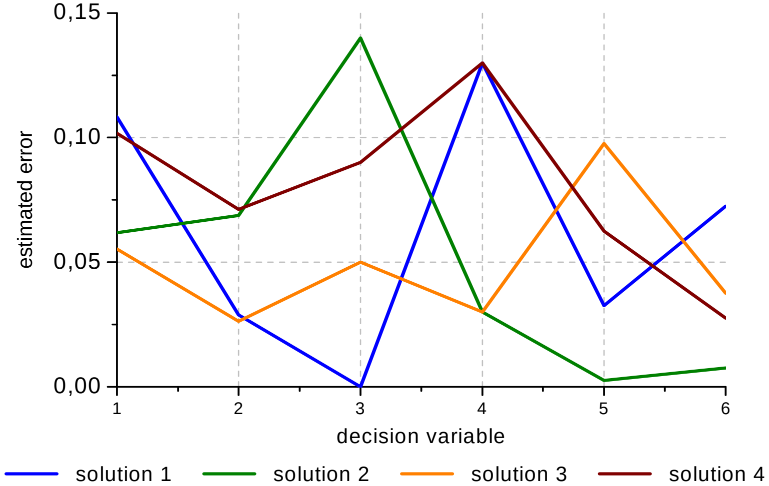
A’posteriori error estimation – an example



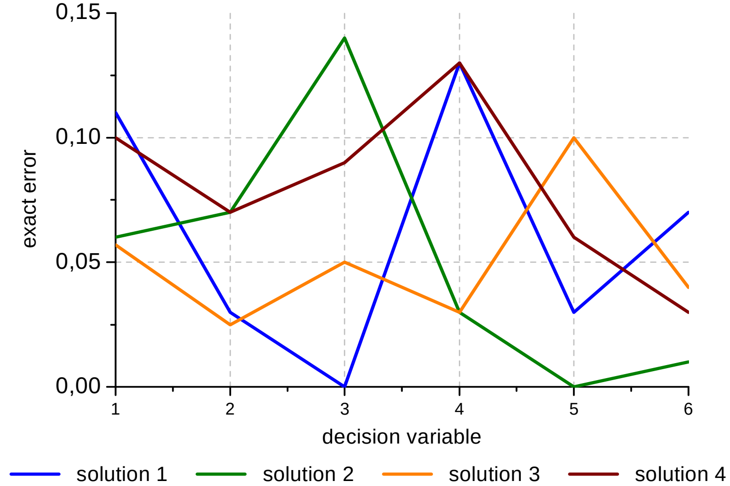
4 independent solutions (6 decision variables) and the exact one



The exact and average solution



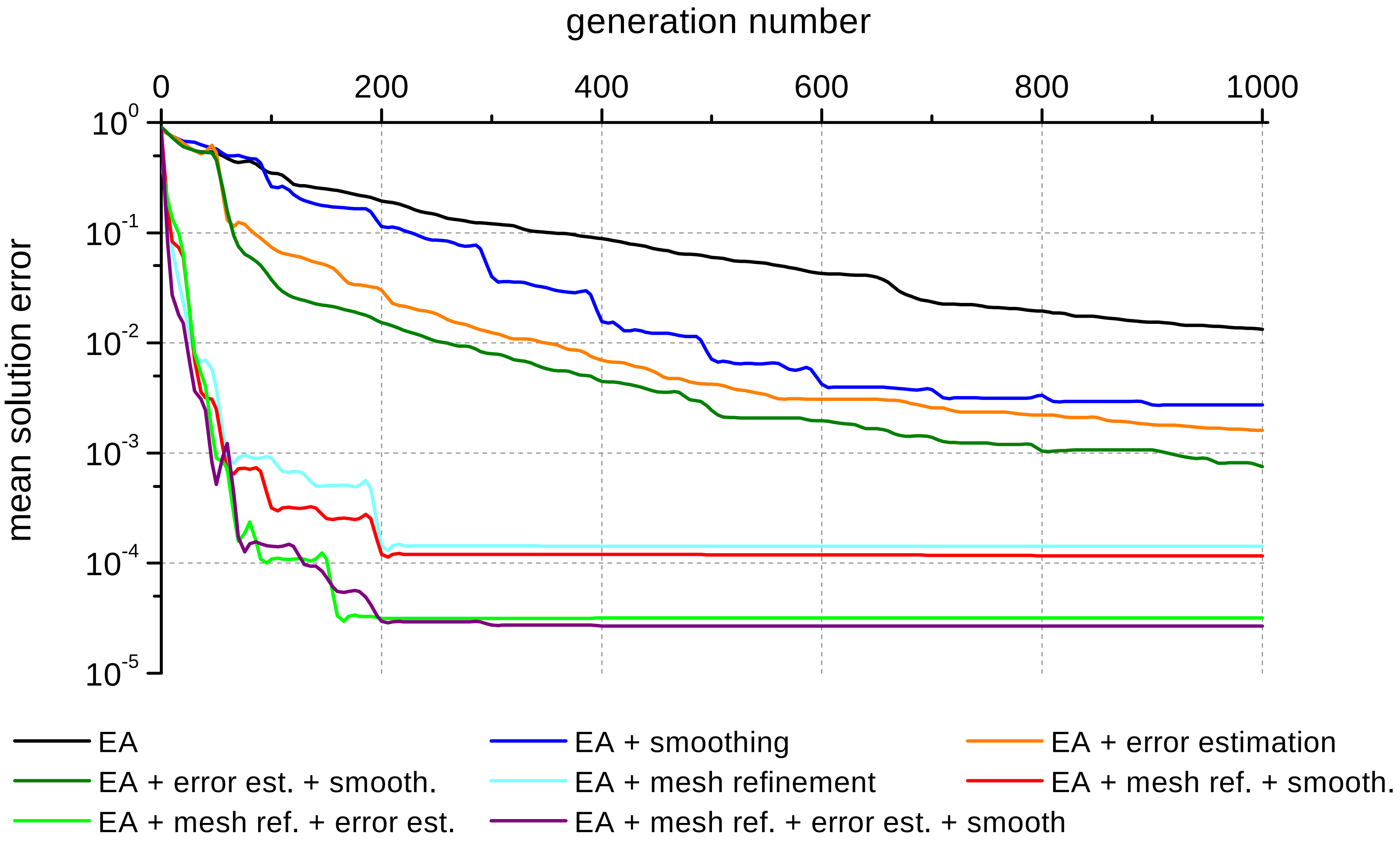
Estimated errors



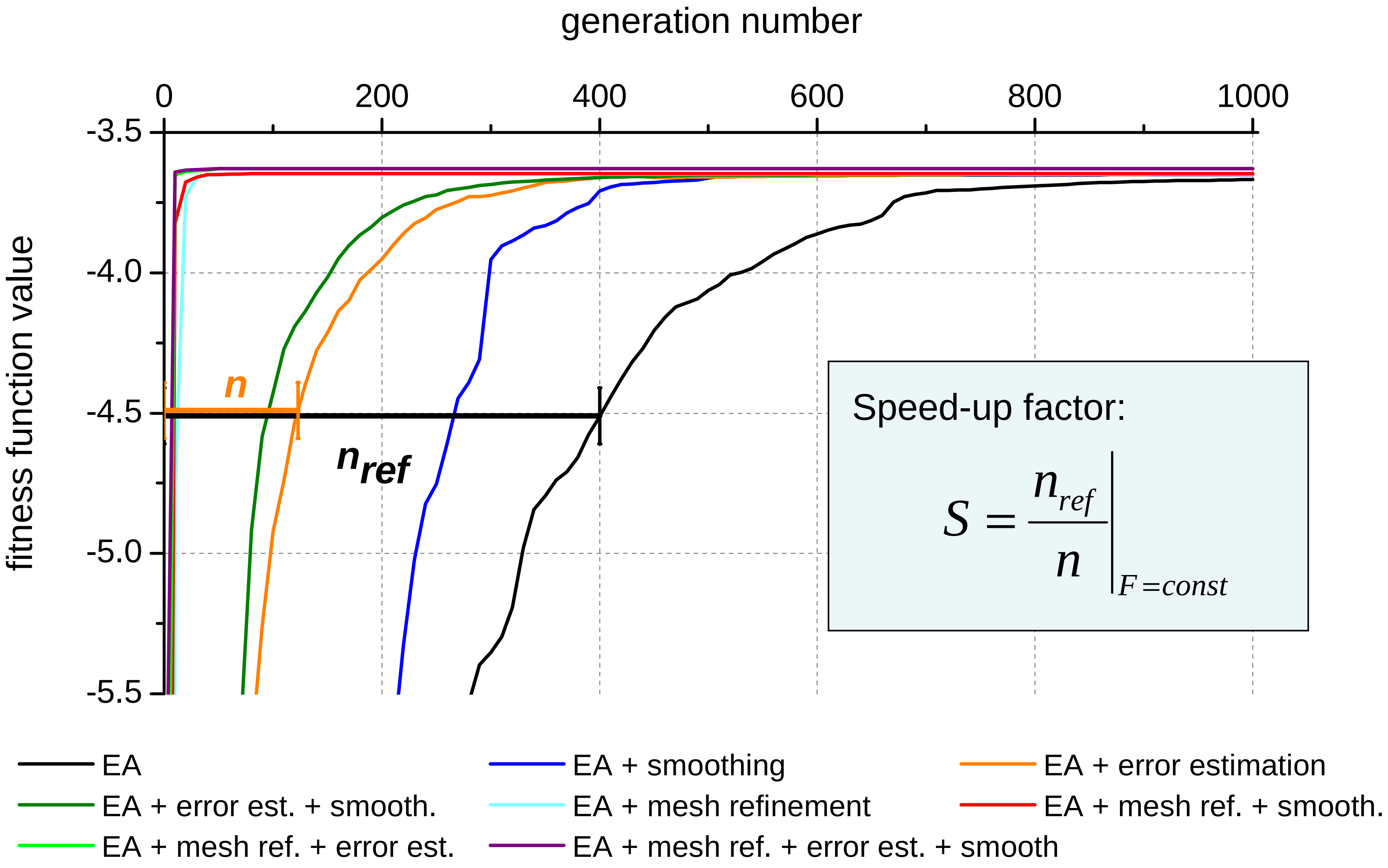
The exact errors

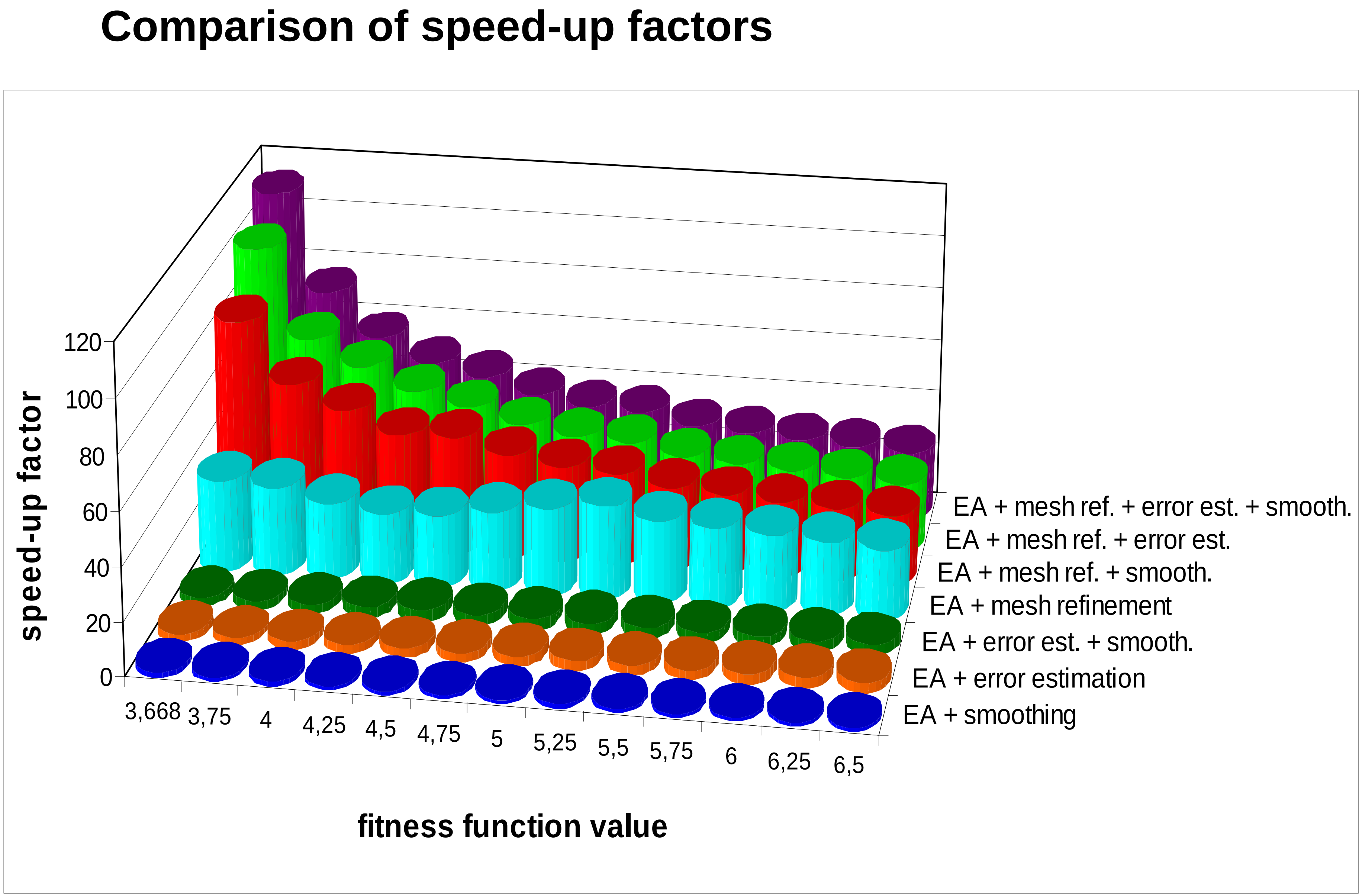
6. RESULTS

Comparison of solution convergence



Comparison of fitness function convergence




Comparison of speed-up factors
speed-up factor
0
20
40
60
80
100
120
3,668
3,75
4
4,25
4,5
4,75
5
5,25
5,5
5,75
6
6,25
6,5
fitness function value
EA + mesh ref. + error est. + smooth.
EA + mesh ref. + error est.
EA + mesh ref. + smooth.
EA + mesh refinement
EA + error est. + smooth.
EA + error estimation
EA + smoothing

7. Final remarks

Summary:

Several attempts have been made in order to speed-up the EA optimization process

(i) The most effective combination of EA operators was searched and examined.

(ii) Several simple concepts, like mesh refinement, a'posteriori error analysis, solution smoothing and balancing, were proposed and investigated in order to accelerate the EA optimization.

(iii) Carefully selected benchmark problems were analysed. The speed-up factor over 100 was obtained.

Further research planned:

- continuation of various efforts oriented towards increasing the EA efficiency;

- analysis of further benchmarks;

- residual stress analysis in railroad rails and vehicle wheels;

- analysis of large, non-linear, constrained optimization problems (convex and non-convex) resulting from the physically based approximation applied to experimental measurements.

Thank you very much for attention

8. References

[1] Burczynski T., Orantek P., *Evolutionary and hybrid algorithms* [in Polish], BEL, Rzeszow, 1999.

[2] Glowacki M., *Effective evolutionary algorithms in physically based smoothing of experimental data* [in Polish], M.Sc. Thesis, CUT, Cracow, 2009.

[3] Gwiazda T., *Genetic algorithms. Reference* [in Polish], Vol. 1, 2, PWN, Warsaw, 2007.

[4] Kleszcz A., *Some attempts to increase effectivity of evolutionary algorithms* [in Polish], M.Sc. Thesis, CUT, Cracow, 2008.

[5] Michalewicz Z., *Genetic Algorithms + Data Structures = Evolution Programs* [in Polish], WNT, Warsaw, 1996.

[6] Orkisz J., Obrzut A., On some attempts of the Evolutionary Algorithms efficiency increase, *9th Conference on Evolutionary Algorithms and Global Optimization KAEiOG*, Murzasichle, 2006.