Ultracold atoms in optical lattice

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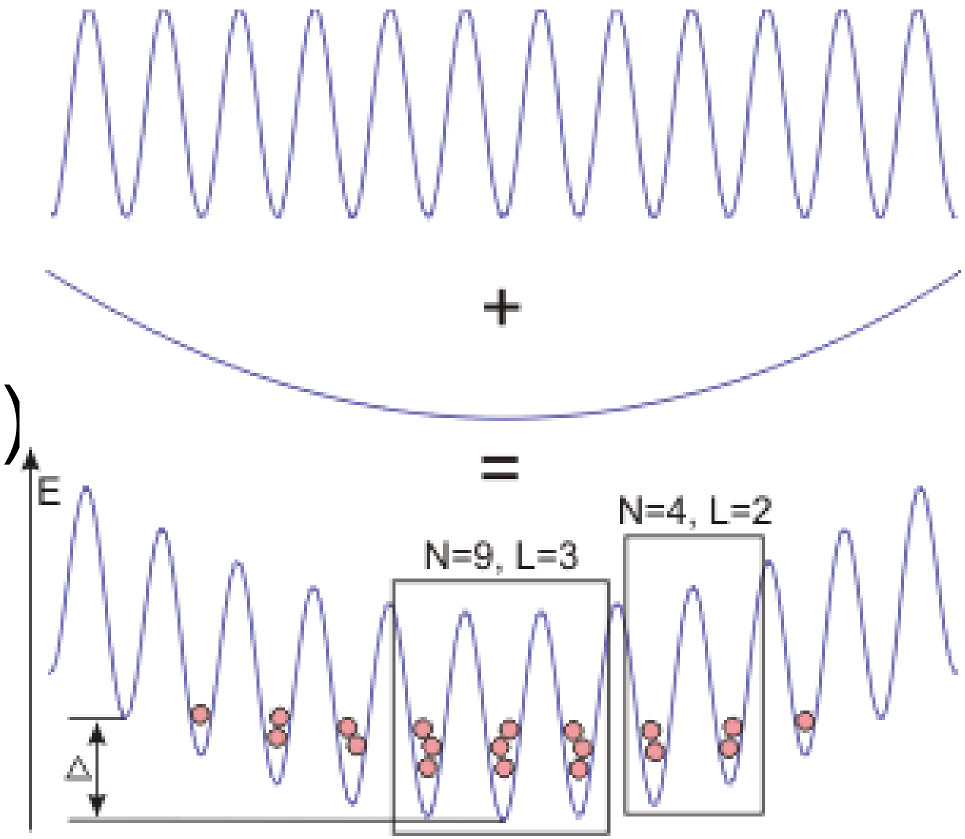
Ultracold atoms

- Optical lattice potential
- Ultralow temperature (nanokelvins)
- Dilute gas (10^{-6} times more dilute than air here)



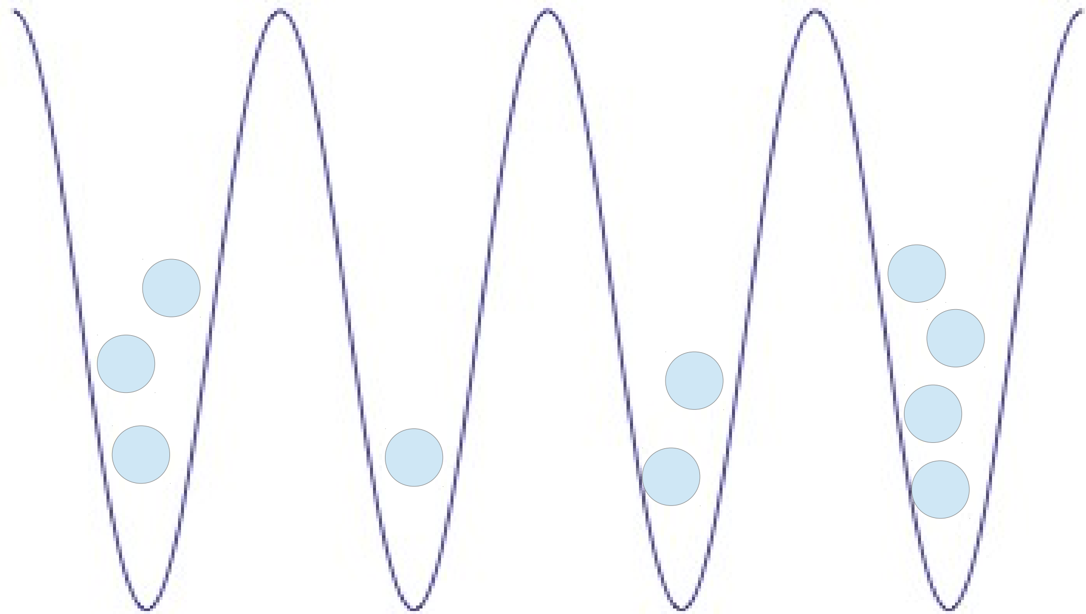
Applications:

- Study of condensed matter hamiltonians: Hubbard, spin...
- No phonons
- Tunability of “material constants”
- Fermions, bosons, anyons
- Particle in a nonableian gauge field
- Long and short range interactions

Hilbert space

Fock states:

Quantum tunneling: Configurations span vector space



$$|3,1,2,4\rangle$$

151 particles, 80 lattice sites, max 5 particles, dimension of the H.S :

$$2025128245687739667575303371494865939109438242486067520 \approx 2 \cdot 10^{54}$$

Low energy states: often also **exponentially** many

Solution: use Matrix Product States

MPS representation

$$|\psi\rangle = \underbrace{\ldots + c_{3124}|3124\rangle + \ldots}_{2\cdot 10^{54}\ \text{summands}}$$

For each site we keep for example 6 matrices:

A_0^1	A_0^2	A_0^3	A_0^4
A_1^1	A_1^2	A_1^3	A_1^4
A_2^1	A_2^2	A_2^3	A_2^4
A_3^1	A_3^2	A_3^3	A_3^4
A_4^1	A_4^2	A_4^3	A_4^4
A_5^1	A_5^2	A_5^3	A_5^4
3	1	2	4

$$c_{3124} = A_3^1 A_1^2 A_2^3 A_4^4$$

Exponential many coefficients

Formed out of **polynomially** many matrices

Bose-Hubbard hamiltonian

$$|3, 1, 2, 4\rangle$$

- N particles in one site, interaction energy $\frac{U}{2}N(N-1)$
- Kinetic energy: proportional to J

$$H = -J\sum_i b_i^\dagger b_{i+1} + \mathrm{h.c} + \frac{U}{2}\hat{n}_i(\hat{n}_i - 1) + \sum_i (V_i - \mu)\hat{n}_i$$

Fourier transform in MPS space

$c_{3124} = A^1_3 A^2_1 A^3_2 A^4_4$ Nonlinear relation between A and ψ = nontrivial addition

Unitary evolution, time independent hamiltonian

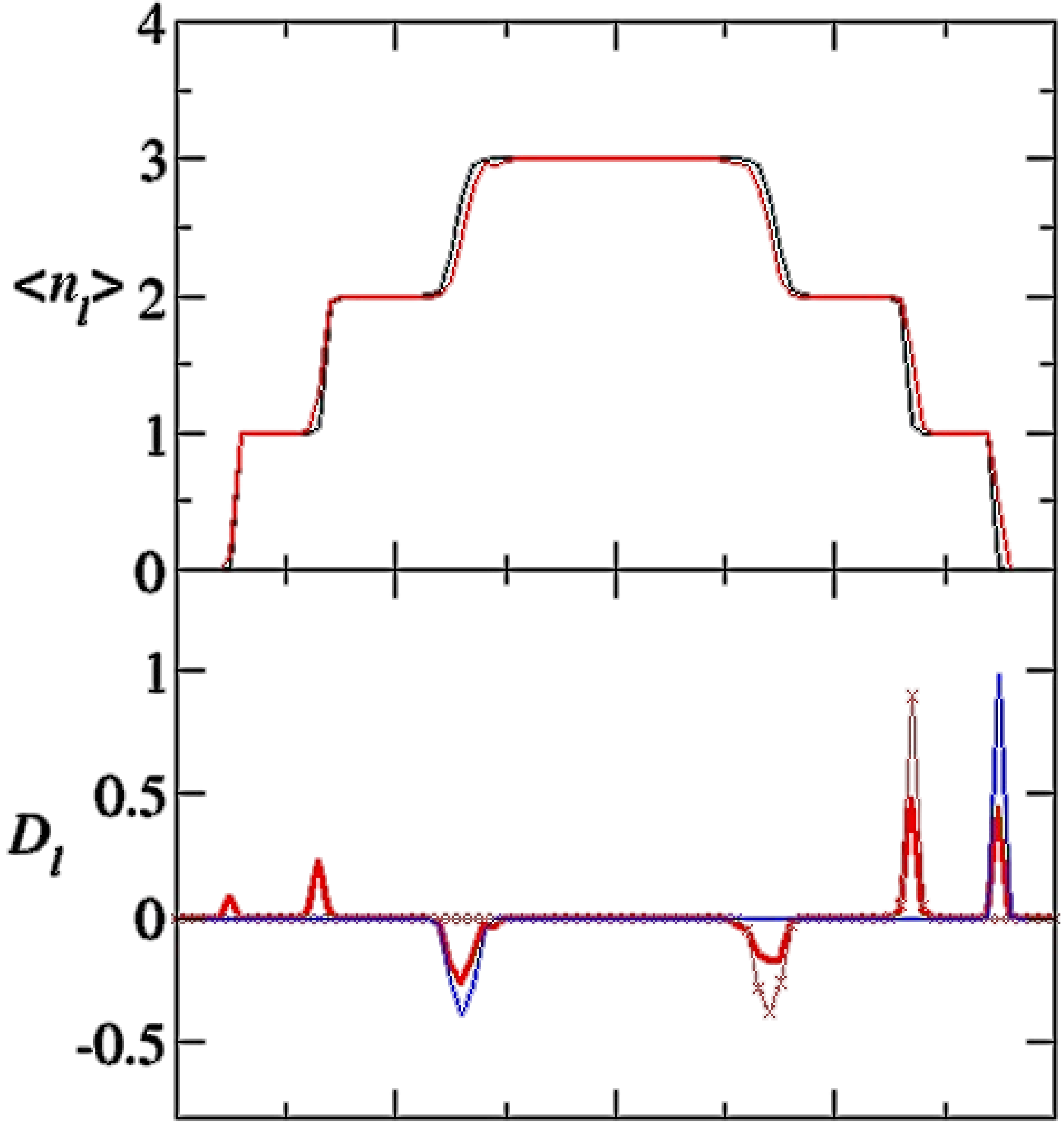
$$\psi(t) = \exp\left(-\frac{i}{\hbar}\mathcal{H}t\right)\psi(0)$$

$$\int_0^\infty dt \exp\left(-\frac{i}{\hbar}\mathcal{H}t\right)\exp\left(\frac{i}{h}Et\right) = P_E$$

MŁ, Dominique Delande, Jakub Zakrzewski, Acta Physica Polonica 12/2011 (2012)

MŁ, Dominique Delande, Jakub Zakrzewski, Phys. Rev. A 86, 013602 (2012)

Eigenstates gallery



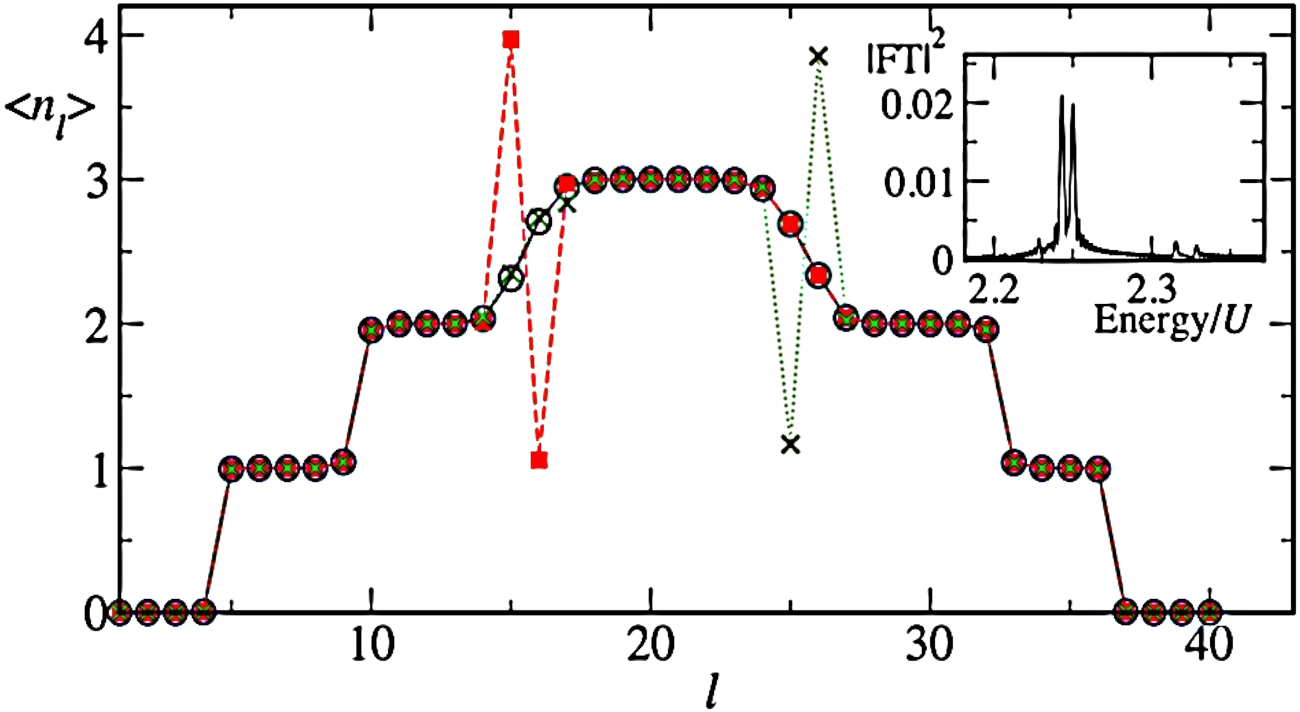
Slow increase of U/J

Long range excitations

MŁ, D. Delande, J. Zakrzewski,
Acta Physica Polonica 12/2011 (2012)

MŁ, D. Delande, J. Zakrzewski,
Phys. Rev. A 86, 013602 (2012)

Eigenstates gallery



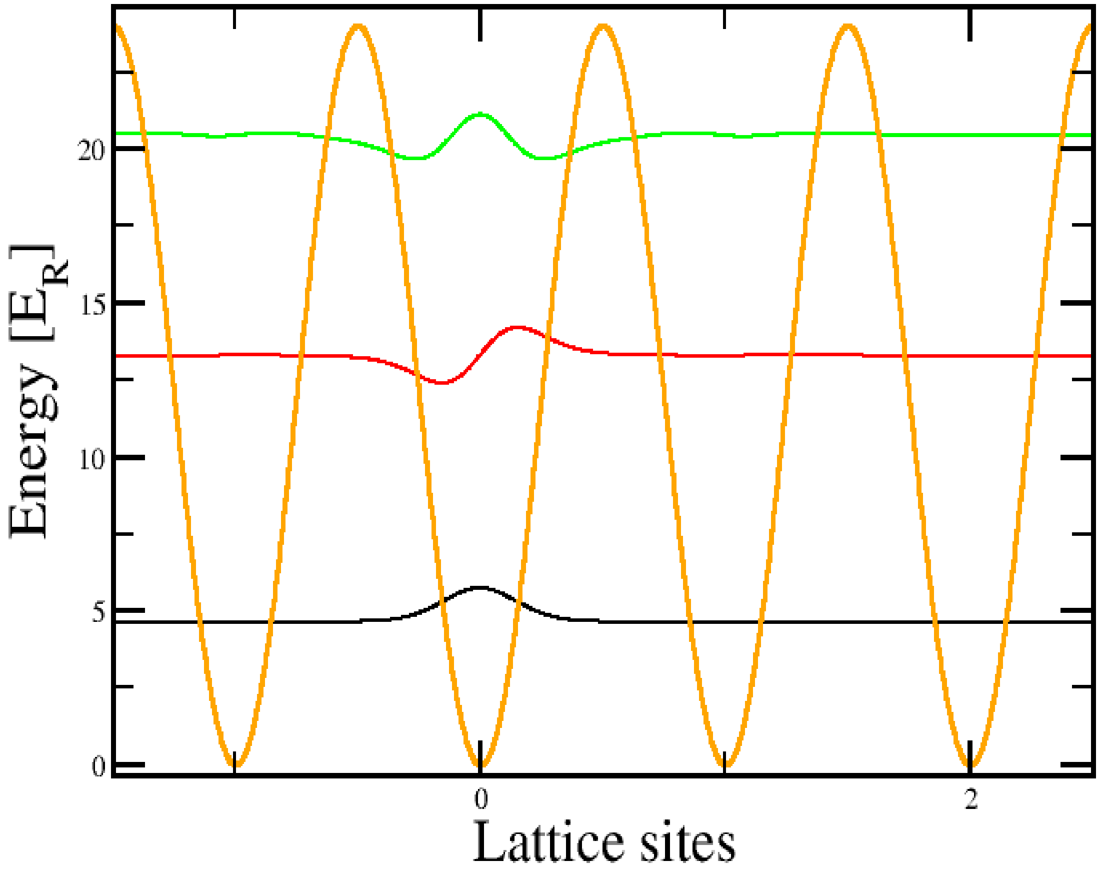
$U/J \sim \sin(\omega t)$

MŁ, D. Delande, J. Zakrzewski, Acta Physica Polonica 12/2011 (2012)

MŁ, D. Delande, J. Zakrzewski, Phys. Rev. A 86, 013602 (2012)

Multiband effects

Previous considerations assumed expansion in functions from first Bloch band only



Multiband effects

Problem : n interacting particles in one lattice site

Description by first 15 Bloch bands

Dirty functions

Instead of Fock Basis

$|n\rangle$ We get new basis out of basis states: $|n\rangle_{MO}$

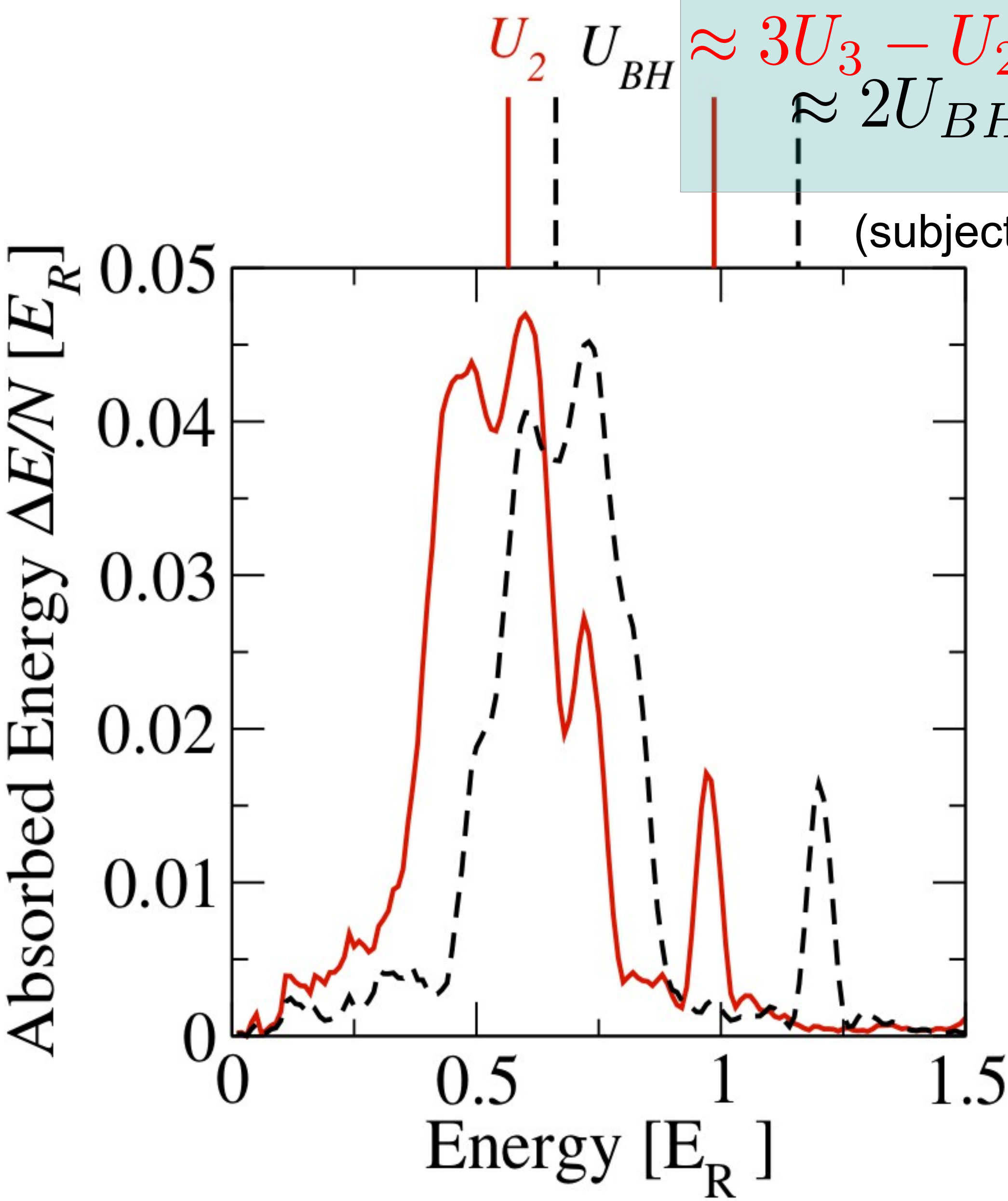
U Becomes density dependent

$\frac{U_n}{2}n(n-1)$ Instead of $\frac{U}{2}n(n-1)$

U_n decreases n

Analogously J_{n_1,n_2} Increases with n

Multiband effects



$\approx 3U_3 - U_2$

$\approx 2U_{BH}$

(subject to certain restrictions)

MŁ, D.Delande, J. Zarzewski, New Journal of Physics 15, 013062 (2013)

Multiband effects

Couplings to higher Bloch bands may be also introduced by the dynamics:

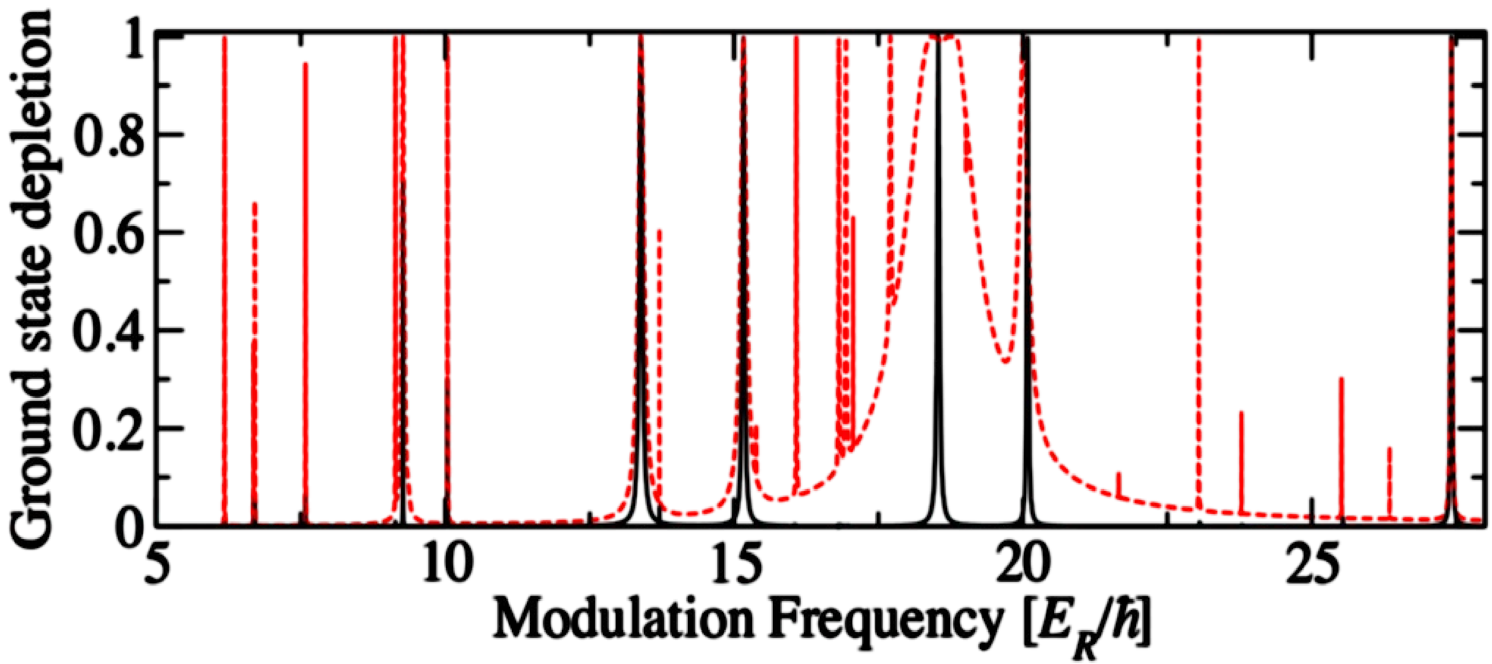
Static lattice:

$$\langle w^{\alpha} | w^{\beta} \rangle = 0 \quad \alpha \neq \beta$$

Lattice potential changes in time:

$$\langle w^{\alpha} | \dot{w}^{\beta} \rangle \neq 0 \quad \Longrightarrow$$

Extra (dynamical) couplings



MŁ, J. Zakrzewski Phys. Rev. Lett. 110, 065301 (2013)

Thank you for attention