1. Describe how to iteratively obtain nonlinear least squares estimates of the parameters of the model:
\[ y_t = \beta_1 x_t^{\beta_2} + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2), t = 1,2,\ldots,T \]
How to obtain reasonable starting values for \( \beta_1, \beta_2 \) using a corresponding data set?
How to calculate (approximate) estimation standard errors?

2. Estimate \( \theta \) in the model:
\[ y_t = t^\theta + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2), \]
using the nonlinear least squares estimator calculated iteratively from the starting point \( \theta^{(0)} = 1.492 \). Obtain both the NLS estimate and its approximate standard error. The data are as follows:

<table>
<thead>
<tr>
<th>( t )</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y_t )</td>
<td>1.19</td>
<td>2.70</td>
<td>5.22</td>
<td>8.00</td>
<td>11.04</td>
</tr>
</tbody>
</table>

Compute the approximate 95\% confidence interval for \( \theta \).

3. Consider the following model:
\[ z_t = \beta_1 + \beta_2 P_t + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \sigma^2), \text{where} \]
\( z_t \) is the quantity of goods sold by a monopolist firm, \( P_t \) is the price level. Historical records of the two variables are given in the table. In the next period the price will be set to 5.

<table>
<thead>
<tr>
<th>( P_t )</th>
<th>2</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>( z_t )</td>
<td>10</td>
<td>12</td>
<td>9</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

- Estimate parameters of the model (together with standard errors of estimation)
- Compute the 95\% confidence intervals for for \( \beta_1, \beta_2 \).
- Test the significance of the parameters (using the significance level 0.05)
- Verify the hypothesis that \( \beta_2 = 2 \) (using the same significance level)
- Compute the 95\% confidence interval for the parameter \( \phi = 2\beta_1 - 3\beta_2 \).
- Forecast the quantity sold in the next period (together with the forecast standard error)

4. On the basis of the data from \( T = 8 \) firms (using the same technology), the following relationship between the capital (K) and labor (L) inputs and the production output (Q) has been estimated:
\[ \ln \hat{Q}_t = 0.85 + 0.81 \ln K_t + 0.48 \ln L_t \]
additional data are given in the table. Assuming normality or the error terms:
- Name the model.
- Interpret the parameters’ estimates (except for the intercept).
- Verify the hypothesis of the constant returns to scale. (assume \( \alpha = 0.05 \))
- Compute the 95\% confidence interval for returns to scale.
- Verify that the elasticity of production output with respect to capital input is equal to the elasticity with respect to labor input.