Tadeusz Sozański

Jagiellonian University

USER'S GUIDE TO NETAID

- 1. Introduction
- 2. Environment you need to run NETAID
- 3. Main Menu. Input and output files
- 4. Graph-theoretic Power Index
- 5. Equidependence iterative procedures
- 6. Analysis of exclusionary power
- 7. Bibliography

September 2003

1. INTRODUCTION

I wrote NETAID to help myself and fellow mathematical sociologists to analyze **power in one-exchange networks**. The program is now offered free to all interested in **Network Exchange Theory** (NET).

An **exchange network** is a social system such that the actors occupying positions in it can gain valued resources only by concluding two-party **transactions** with their 'neighbors' in a communication network which is mathematically modeled as a **connected graph**. In a **one-exchange network**, the interdependence of dyads in the system is induced by the **one-exchange rule** which permits each actor to make only one deal in a **round** (a stage of the networkwide negotiation process).

The one-exchange rule determines the **family of transaction sets**. Each transaction set, or a **matching** (the term used in graph theory) is a collection of lines such that no two of them have a common endpoint. A transaction set is called **maximal** if it cannot be extended by a line without violating the one-exchange rule.

The program's welcome screen shows the **complete one-exchange network** with 5 positions. Next to the **adjacency matrix** of the transaction opportunity graph you can see the graph's **geometric representation**. Two lines which form one of 15 maximal transaction sets are drawn in blue. Four positions which are 'included in a transaction' have blue labels; the fifth position with red label is 'excluded.'

In general, we say that **position P can exclude position Q** if there exists a maximal transaction set T such that P is an endpoint of a line in T ('P is included in a transaction in T), and Q is not an endpoint of any line in T ('P is excluded from transactions in T).

My interest in theory and research in NET dates back to 1990 when I became affiliated with the Chair of Group Processes headed by Jacek Szmatka. In Spring 1993 I read the **special issue of 'Social Networks'** (vol. 14, no. 3-4, 1992) on NET, edited by David Willer, where I found inspiration for my own mathematical research on exchange networks.

The first version of NETAID written in **1993** had only a procedure for computing **Graph-theoretic Power Index** defined by Markovsky, Willer, and Patton (1988). The second version of my program was attached with my **1997** paper in which I offered a mathematical elaboration of the **Equidependence Theory** of Cook and Yamagishi (1992). The aim of the second procedure then added to NETAID was to examine convergence of the algorithm I proposed as a method to determine an alternative solution of Cook and Yamagishi's equidependence equation.

The third version of NETAID I completed in Summer **2001** provides tools for the **analysis of exclusionary power** which is one of the main topics of my book currently in process. The third procedure (see Section 6) determines four elementary relations, lists all maximal matchings, and computes five power parameters, including the one known as 'exchange-seek likelihood of inclusion' (see Markovsky et al. 1993). The next version of NETAID (October **2002**) determines, in addition, the type (strong vs. weak) of elementary power. The latest version contains the original iterative algorithm proposed by Cook and Yamagishi so that a comparison of two equidependence theories is now possible.

2. 'HARD' AND 'SOFT' ENVIRONMENT YOU NEED TO RUN NETAID

I wrote NETAID in **Quick Basic 4.5** and compiled the source code as the main program NETAID.EXE which calls 3 compiled subprograms (files NETSUB1.EXE, NETSUB2.EXE, and NETSUB3.EXE). File BRUN45.EXE must reside in the current directory to establish communication within the **chained**

program package. NETAID runs best under DOS 5.0 or any later version; if your PC uses Windows 95 or 98 as operating system, switch to the MSDOS mode.

NETAID can process both 'small' (2-6 nodes) and 'large' (7-8 nodes) one-exchange networks. 9-node networks are also accepted, but 3rd NET subroutine will be completed only if the number of transaction sets does not exceed 764, which is the number found for the complete 8-node network.

Use the fastest computer available. If you embark on analyzing exclusionary power in 'large' and 'thick' networks, and have at best a 386 or 486 processor, you will have to wait many hours for the results. I tested NETAID on a PC equipped with a Pentium III processor with a 733 Mhz clock. To compute the most sophisticated power parameter for the 6-, 7- and 8-node complete graphs that computer needed 18 seconds, 20 minutes, and 5¹/₄ hours, respectively.

At last, you should have a VGA graphic card and a color monitor in order to see network lines drawn in colors on the Quick Basic Screen 9 (80x25 characters in text mode, 640x350 points in graphic mode)

3. MAIN MENU. INPUT AND OUTPUT FILES

NETAID is intended to be user-friendly through extensive use of menus and on-screen instructions. To select an item from a menu just type its number. Type **Y** or **N** to answer yes-or-no questions (lowercase and uppercase letter are never distinguished). Press the **space key** when you are prompted to do so in order to move on. The **Enter key** is used to confirm keyboard input only if strings to be typed may vary in length.

In filenames, only letters, figures and the underline character are accepted; the name length may not exceed 8 characters. Input files must not have extensions while output files receive extension TXT automatically.

You start from the **NETAID Main Menu** which allows you to choose the source of data passed to the program's procedures. When you pick out option #1, you are asked to enter the name of a **graph source file**. Such an ASCII text file is structured as a sequence of lines; each **input line** containing all information needed to define one graph must begin from the **number of nodes** *n*. If a figure from the range 2-9 is not found in there, the line is skipped as a **comment line**.

The sequence of two-character **node labels** is recorded in the next 2*n* columns. The first element of a label must be a letter; the second can be space, a letter, a figure or apostrophe. (If the initial letter is not found for at least one node, NETAID will automatically label nodes 1,2,...,*n* as A,B,...). The next $\frac{1}{2}(n(n-1))$ columns are reserved for the above-diagonal entries of the **adjacency matrix** listed row by row from top to bottom.

The column past the 0-1 string is the last column read by NETAID. It may contain a space (the default) or a **letter which marks the graph**. Marks can be added, changed or removed during on-screen display of graphs (you can save the changes in a new source file). The graphs marked with the same letter can be later retrieved for display. If no mark is found, there remain **two modes of searching the graph list**.

First, graphs can be listed one by one down from the number given by the user. If you choose search mode #2, you will be asked every time to enter the **reference number** of a graph to be displayed. Reference numbers are assigned to the graphs by the program which counts input lines in the file and reads each to check its contents for consistency. To make it easier to prepare a properly structured file I endowed NETAID with a **graph definition procedure** (option #3 in the Main Menu) which allows to edit a new source file or append more lines to an already existing file. The same subroutine is used to **define a graph ad hoc**. You can use this option (#2 in the Main Menu) when you want to examine a single network you will not need to save for further study.

To define a graph, you must enter the number of its nodes, first. Next you are prompted to attach labels with the nodes numbered from 1 to n. The next step is to fill out the 'upperhalf' of the adjacency

matrix. As soon as you put 1 in an *ij* cell, the line joining *i*th and *j*th point is added to the picture drawn on the screen at the same time. The display of graphs in NETAID is based on depicting nodes as **vertices of a regular polygon** (the vertices are numbered counterclockwise).

NETAID can read a file with at most 12500 input lines. The contents of the graph source file is rewritten to a **temporary random access file** (extension TMP is added to the name of the source file) from which records are selected to be processed by NET procedures.

To make possible systematic study of a large class of one-exchange networks I prepared a special source file called CGRAPHS containing all **995 non-isomorphic connected graphs with 2-7 nodes**. The list of their adjacency matrices comes from the Appendix to a paper by John Skvoretz (1996). I reshuffled the node numbers in each graph and transformed its adjacency matrix accordingly so as to ensure that the segments corresponding to graph lines under the polygon geometric representation intersect at a minimum number of points.

By means of a special program I generated also a **structural labeling** of positions for each network. Automorphically related nodes are then assigned the same letter; if there are at least two nodes in one class, successive numbers are appended to the letter. In addition, letters A,B,... have been selected in such a way that the alphabetic order of labels agrees with the ranking of nodes with respect to decreasing **degree** (the number of 'neighbors' of a node).

Once you've passed to NETAID your decision on data input, the program will ask you whether the results of NET procedures are to be saved in a file. To give a negative answer, just press Enter at the filename prompt.

An **output file** produced by NETAID is a simple ASCII text file (the extension TXT is added by the program) whose lines contain only characters available from the standard keyboard. The file, by its content and structure, basically reproduces the text data displayed on the screen. However, in some cases the numbers shown in a rounded form are saved in the output file with greater accuracy.

4. GRAPH-THEORETIC POWER INDEX

To compute **Graph-theoretic Power Index** (*GPI*) for *i*th point according to the definition given by Markovsky, Willer and Patton (1988) one needs to find – for *k* from 1 through the length of the longest simple chain in this graph – the **maximum number of pairwise point-disjoint simple chains of length k going out of ith point**. *GPI* is obtained by adding up these numbers over odd *k* and subtracting from the result the similar sum found for even *k*.

In Markovsky, Willer and Patton's theory, *GPI* not only generates an ordering of network positions with respect to 'power'; it is also used to define the **exchange-seek relation**. The current version of NETAID implements the definition of this relation modified by Lovaglia et al. (1999). It is based on classifying any neighbor Q of P as weaker than P [*GPI*(Q)<*GPI*(P)], equal in power to P [*GPI*(Q)=*GPI*(P)], or stronger than P [*GPI*(Q)>*GPI*(P)]. Then P seeks exchange only with all Qs in the weakest of all nonempty categories of its neighbors.

NETAID determines the exchange-seek relation so defined and identifies three cases possible for each line: reciprocated partner choices, unilateral choice, and mutual no-choice. The three cases are marked in the geometric representation of the network by **green**, white, and red line color, respectively. In Markovsky et al.'s theory the latter two cases are referred to as **network break lines**.

If such a line is found, NETAID does the **Iterative GPI Analysis** (Markovsky et al. 1993). *GPI* values are recalculated for all nodes of the graph stripped of all red and white lines, and the exchange-seek relation is determined again from the new values but the removed lines remain in the graph. If a network break line is found again, the procedure is repeated, and continued until the current *GPI* values coincide with those obtained at an earlier step. In some cases, the state once attained repeats at the next step and becomes the final state. However, for many graphs, the iterative procedure results in reaching a state which opens a sequence of two or more 'solutions' that are since then

cyclically repeated infinitely many times. NETAID stops *GPI* analysis when first such cycle is closed. For 9-node networks the process continues until the number of steps attains the maximum of 31 found for all connected graphs with 2-8 nodes.

The paper by Lovaglia et al. (1999) describes an iterative procedure which can be applied when Markovsky et al.'s algorithm leads to a repeating cycle of solutions. The new procedure is not available in NETAID (a program written by John Skvoretz is mentioned in the cited paper).

5. EQUIDEPENDENCE ITERATIVE PROCEDURES

Cook and Yamagishi's (1992) theory stemmed from Emerson's idea of equal dependence of exchange partners. Formally, it is the theory of a certain **dynamical system** associated with a one-exchange network. The **space of states** of this system is made up of all $n \times n$ matrices R such that $R_{ij}+R_{ij}=C$ if positions *i* and *j* are connected in the network, and $R_{ij}=R_{ji}=0$ otherwise. R_{ij} is interpreted as the share that actor *i* currently hopes to gain in a transaction with actor *j*. A transaction consists in splitting the pool of C 'profit points' between *i* and *j*.

The *ij* entry of **alternative profit matrix** A(R) is defined for tied *i* and *j* as the maximum of R_{ik} for *k* other than *j*; $A(R)_{ij}=0$ if *i* and *j* are not connected. D(R)=R-A(R) is called the **dependence matrix**. The 'principle of equal dependence' claims that the networkwide bargaining ends when $D(R)_{ij}=D(R)_{ji}$ for any network line *ij*, which means that the final *R* must satisfy the **equidependence equation** $D(R)=D(R)^{t}$, or $R-A(R)=R^{t}-A(R)^{t}$ where ^t is used to mark the **transpose** of a matrix.

Cook and Yamagishi (1992) offered also a method to find a solution of this equation, but their iterative procedure does not guarantee that the matrix so obtained satisfies for any *ij* the **self-complementarity condition** $R_{ij}+R_{ji}=C$. In my 1997 paper, I showed how to correct this 'flaw' which, as I see it now, is not a flaw but an alternative theoretical option (see the end of this section). First, I noticed that the equidependence equation can be rewritten in the form T(R)=R where *T* is the **transformation of the space of states into itself** which assigns $\frac{1}{2}(C+A(R)-A(R)^{t})$ to *R*. *T* is the composition of two mappings: A: $R \rightarrow A(R)$ and *U*: $S - \frac{1}{2}(C+S-S^{t})$. Mapping *A* defines new profit claims addressed by *i* and *j* to each other as the maximum payoffs they could obtain from transactions with alternative partners. The new claims $A(R)_{ij}$ and $A(R)_{ji}$ which need not sum up to *C* are adjusted by the mapping *U* which restores complementarity, assumed for initial claims R_{ij} and R_{ji} . As a consequence, T maps the set of self-complementary *R*-matrices into itself, and every *R* satisfying the equidependence equation and self-complementarity condition is a **fixed point** of *T*.

Having proved that *T* must always have a fixed point, I solved the equation T(R)=R with the constraint $R+R^{t}=C$ for all connected graphs with 2–6 nodes. For larger graphs the task becomes too laborious, so that an iterative procedure is needed. The algorithm offered in NETAID is based on the following theorem: if the sequence $T^{k}(R_{0})$, where T^{k} is *k*th power of *T* and R_{0} is an initial matrix, converges to a matrix *R*, then *R* is a solution of the equation T(R)=R. If the solution is known, one can examine its **reachability** from varying initial states.

The equidependence iterative procedure programmed in NETAID applies only to **homogenous one-exchange networks** in which every line of the transaction opportunity graph is assigned a profit pool of the same size, set in NETAID to 100 points. The entries of R_0 can be typed from keyboard or filled with random values; the third item in the menu is setting the 50-50 pool split in every line.

The iteration process stops after the number of steps entered by the the user (1000 is the maximum allowed). The **stop condition** can also be imposed by setting an upper bound on max $|T^{k}(R_{0})_{ij}-T^{k-1}(R_{0})_{ji}|$; then the process will end as soon as the distance between last two iterations does not exceed the given threshold, say, .01 (.0001 is the minimum accepted).

The value of **Equidependence Power Index** for *i*th position was defined by Cook and Yamagishi (1992) as max R_{ij} over all *j*. The EPI values, computed by NETAID for the initial and final *R*, are recalculated for *C*=24 (the pool size used in experiments) and displayed in the rounded form at the vertices of the graph's geometric representation. **Green and red lines** show all reciprocated positive

and negative exchange seeks. I defined the **exchange-seek relation generated by R** by the condition: *i* seeks exchange with *j* if and only R_{ij} =*EPI*(R)_{*j*}.

In 2002 I noticed that Cook and Yamagishi's algorithm can also be expressed in terms of a transformation whose successive powers are calculated for a given initial matrix R_0 . Let T be the composition of $A: R \rightarrow A(R)$ and the mapping $U: S \rightarrow U'(S)$ such that $U'(S)_{ij} = U(S)_{ij}$, $U'(S)_{jj} = U(S)_{jj}$ for every network line *ij* for which $S_{ij}+S_{ji} \leq C$, and $U'(S)_{ij}=C-S_{ji}$, $U'(S)_{ij}=C-S_{ij}$ for every network line *ij* for which $S_{ij}+S_{ji} \geq C$. Consider the set of R-matrices such that $R_{ij}+R_{jj} \leq C$ for every network line *ij*. Clearly, T transforms this set into itself. **Cook and Yamagishi's equidependence solution** can be defined analytically as a fixed point of T. Both solutions, the one given in my 1997 paper and Cook and Yamagishi's solution, satisfy the equidependence equation. They share many properties, in particular, both generate symmetric exchange-seek relations. A very important difference between two variants of equidependence theory is that Cook and Yamagishi's theory sometimes attributes equal power to two positions P and Q such that P is the only available partner for Q, but P has other partners besides Q. In my variant of equidependence theory, *EPI* always points to the advantage of P over its **hanging** neighbor Q.

6. ANALYSIS OF EXCLUSIONARY POWER

Willer and Markovsky (1993: 340) claim that 'if a structure permits one set of actors to exclude a subset of the actors to which they are connected which cannot exclude in return, then the first set develops power over the second.' The concept of **power** is formalized next by means of structural parameters (GPI, ESL). Since 1991 the problem of how to formally distinguish 'weak' from 'strong' power has pervaded the Elementary Theory variety of NET. Recent papers (Lovaglia et al. 1999, Simpson and Willer 1999) take up this intriguing issue once again. The intention to save as much as possible from earlier theory results in that the procedures proposed to solve the problem are very complex. In my book *The Mathematics of Exchange Networks* (in process, to appear in 2005) the concept of exclusionary power is reconsidered so as to make the theory foundations more precise, and, first of all, simpler.

Having already defined the **exclusion relation** (see Introduction) I define in turn the **elementary power relation** by the condition: Position *P* has power over position *Q*, symbolically, P>Q, if and only if *P* can exclude *Q* and *Q* cannot exclude *P*. *P* and *Q* are said to satisfy the **elementary equipower relation** (P~Q) if and only if *P* cannot exclude *Q* and *Q* cannot exclude *P*. Elementary power and its inverse (P<Q iff Q>P) are antisymmetric and transitive relations; the equipower relation is an equivalence (all proofs will be given in my book). The fourth elementary relation possible between two network positions is that of **mutual excludability**: *P* can exclude *Q* and *Q* can exclude *P*.

When you order the 'analysis of exclusionary power,' NETAID will let you see first the $n \times n$ matrix having in any cell one of four symbols: >,<, -, + standing for the four elementary relations. On the right margin are shown the values of two simple power parameters. The first called the **Power Degree** is defined for a position *P* as the number of *Q*s such that *P*>*Q*. The second called the **Power Level** is the length of the longest (directed) path from *P* in the **power digraph** whose arcs are node pairs ordered by the elementary power relation.

In general, the term **structural power parameter** is referred to any mapping *F*, defined on the set of nodes, which assigns the same numerical values to automorphically related nodes and preserves the elementary power and equipower relations (P>Q implies that F(P)>F(Q); P~Q implies that F(P)=F(Q)). A power parameter can be used to **extend the power relation** to pairs of positions which can exclude each other. When such two positions have the same value of *F* they are treated as equally powerful, otherwise one position is considered more powerful than the other.

Given a **probability distribution on the set of maximal transaction sets**, a power parameter can be defined by assigning to *P* the probability that *P* is included in a transaction; the latter probability is obtained by summing the probabilities of all maximal transaction sets covering *P* (*T* covers *P* if $PQ \in T$ for some *Q*). The simplest **probability power index** *PPI1* is obtained by assuming after

Friedkin (1992) that **all maximal transaction sets are equally likely**. Then *P*'s likelihood of inclusion is simply the ratio of the number of maximal sets which cover *P* to the number of all maximal sets.

NETAID computes also two other probability power measures noted *PPI2* and *PPI3*. They are defined in the context of a **Markov chain model** with the set of all transaction sets as the space of states and non-zero **transition probabilities** allowed only from *S* to *S*' where *S* is a subset of *S*'. The empty set is the unique initial state while maximal transaction sets are the only final (absorbing) states. Then, the probability of a maximal set *T* is determined as the probability that T is reached from the initial state in any number of steps.

PPI2 is computed on the assumption that *S* can be extended in one step by only one transaction, and **all transitions from S to its one-line extensions are equally likely**.

The definition of *PPI3* assumes that *S* can be extended in one step to any *S*' with the probability which depends on how frequent are patterns which yield *S*' among all **profiles of partner choices** possible to be made among themselves by the positions not covered by *S*. The computations show that *PPI3* takes the same values as the parameter defined by Markovsky et al. (1993) known as **Exchange-Seek Likelihood** (ESL).

As soon as *PPI1* is found automatically NETAID proceeds to compute *PPI2* or *PPI3* or both according to user's request. Unlike the calculation of *PPI2*, computing *PPI3* for 'thick' graphs may take a lot of time. The probability distribution which has been generated for the computation of the probability power index chosen last is later used to determine the probabilities needed on further steps of the procedure execution.

In the first step, maximal transaction sets and their probabilities are listed and saved in the output file upon request. A **green line** means that two tied positions 'seek exchange' with each other. In the output file, maximal sets with all lines printed in green are marked with *. Unilateral exchange seeks and mutual rejections are displayed in the listing and network picture in **white and red**, respectively.

The definition of **exchange-seek relation** makes use of the elementary relations between *P* and all *Q*s tied to *P*. If all neighbors of P are in the same relation with *P*, *P* seeks exchange with all of them. If *P*>*Q* and *P*~*Q*', then *Q* but not *Q*' is sought by *P*; if *P*<*Q* and *P*(+)*Q*' (mutual excludability), then *P* seeks exchange with *Q*' but not with *Q*.

We say that two positions P and P' are in the same **power component** if they are joined by a chain made up of lines such that two positions which form a line in it seek exchange with each other. This relation is used to define **strong and weak variety of elementary power**. Let us first define a **high power position** as any P such that P>Q for some Q tied to P. The elementary power of P over its neighbor Q is called **strong** if Q seeks exchange with P and P can exclude Q by means of a maximal transaction set T which has the following property: any high power position P' (including P) in the same power component has a neighbor Q' such tha P'Q' is in T and P' and Q' seek exchange with each other.

If the elementary power of *P* over *Q* tied to *P* is not strong, it is called **weak**.

NETAID determines the type of power for any two tied positions. Weak power and strong power dyads are marked on the drawing with arrows $-\rangle$ and $-\bullet$. The elementary equipower relation is shown by placing / across a line. Unmarked lines join positions in the relation of mutual excludability.

At the last step of the analysis, NETAID lists network lines and shows their selected characteristics. **Suboptimal lines** are written in the form P Q instead of P-Q. A line is called suboptimal if it does not occur in any optimal transaction set. **Optimal transactions sets** are those having the maximum number of elements.

For any line there is given first its probability, found by adding up the probabilities of all maximal transaction sets containing the line. The difference $d_{PQ}=PPI(P)-PPI(Q)$ is given in the next column. The order of line endpoints is chosen in such a way that PPI(P) is always greater than or equal to PPI(Q). As a consequence, $d_{PQ} \ge 0$.

The **GPI-R theory** by Lovaglia et al. (1995) implies that $x_{PQ}^-x_{QP}$ equals $\frac{1}{2}Cd_{PQ}$ where x_{PQ} and x_{QP} are shares of *P* and *Q* in the theoretically predicted split of the pool of *C* profit points ($x_{PQ}^+x_{QP}^-C$).

The *GPI-RD* theory offered in the same paper takes into account the degrees of *P* and *Q* in assessing the size of *P*'s power over *Q*. A better solution recently suggested by Lovaglia and Willer (1999) is the **replacing of a PPI by its square**. *P*'s payoffs derived for C=24 from the *GPI-R* theory and its refinement in question are given in columns labeled *x* and *x*'. As the authors stay with ESL, to see the scores they predict, one must request the computation of *PPI3*.

7. BIBLIOGRAPHY

Collections of papers

SOCIAL NETWORKS, vol. 14, 1992, no. 3-4.

Special issue of SN, edited by D. Willer, contains a number of contributions which cover all main approaches in Network Exchange Theory: Elementary Theory (Markovsky, Skvoretz, Willer), Power-Dependence Theory (Cook, Yamagishi), Multi-Person Game Theory (Bienenstock, Bonacich), Expected Value Model (Friedkin).

NETWORK EXCHANGE THEORY. Ed. by D. Willer. 1999.

Westport, Connecticut-London: Praeger.

The book contains old papers reprinted from journals and new contributions; all of them belong only to the Elementary Theory variety of NET.

References

- 1992 COOK Karen S. and Toshio YAMAGISHI. *Power in Exchange Networks: A Power-Dependence Formulation*. SOCIAL NETWORKS 14: 245–265.
- 1992 FRIEDKIN Noah E. An Expected Value Model of Social Power: Predictions for Selected Exchange Networks. SOCIAL NETWORKS 14: 213–229.
- 1995 LOVAGLIA Michael J., John SKVORETZ, David WILLER, and Barry MARKOVSKY. *Negotiated Exchanges in Social Networks*. SOCIAL FORCES 74: 123–155.
- 1999 LOVAGLIA Michael J., John SKVORETZ, Barry MARKOVSKY, and David WILLER. *An Automated Approach to the Theoretical Analysis of Difficult Problems*. Pp. 259–269 in NETWORK EXCHANGE THEORY. Ed. by D. Willer.
- 1999 LOVAGLIA Michael J., David WILLER. *li2, An Alternative for Predicting Weak Power*. Pp. 185–191 in NETWORK EXCHANGE THEORY. Ed. by D. Willer.
- 1993 MARKOVSKY Barry, John SKVORETZ, David WILLER, Michael J. LOVAGLIA, and Jeffrey ERGER. *The Seeds of Weak Power: An Extension of Network Exchange Theory*. AMERICAN SOCIOLOGICAL REVIEW 58: 197–209.
- 1988 MARKOVSKY Barry, David WILLER and Travis PATTON. *Power Relations in Exchange Networks*. AMERICAN SOCIOLOGICAL REVIEW 53: 220–236.

- 1999 SIMPSON Brent and David WILLER. A New Method for Finding Power Structures. Pp. 270–284 in NETWORK EXCHANGE THEORY. Ed. by D. Willer.
- 1996 SKVORETZ John. An Algorithm to Generate Connected Graphs. CURRENT RESEARCH IN SOCIAL PSYCHOLOGY 1 no. 5. http://www.uiowa.edu/~grpproc/ crisp/
- 1997 SOZAŃSKI Tadeusz. *Toward a Formal Theory of Equilibrium in Network Exchange Systems*. Pp. 303–350 in STATUS, NETWORK, AND STRUCTURE. THEORY DEVELOPMENT IN GROUP PROCESSES. Ed. by Jacek Szmatka, John Skvoretz, and Joseph Berger. Stanford: Stanford University Press.
- 1993 WILLER David and Barry MARKOVSKY. *Elementary Theory: Its Development and Research Program.* Pp. 323–363 in THEORETICAL RESEARCH PROGRAMS: STUDIES IN THE GROWTH OF THEORY. Ed. by Joseph Berger and Morris Zelditch, Jr. Stanford: Stanford University Press.

Updated May 2004 Tadeusz Sozański

Http://www.cyf-kr.edu.pl/~ussozans/